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Third Semester B.E. Degree Examination, December 2012
Discrete Mathematical Structures

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting at least TWO questions from each part.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

PART – A

1 a. For any sets A, B, C, D prove by using the laws that
$$(A \cap B) \cup (A \cap B \cap \overline{C} \cap D) \cup (\overline{A} \cap B) = B$$ **(06 Marks)**

b. If $S, T \subseteq U$, prove that S and T are disjoint if and only if $S \cup T = S \Delta T$ **(04 Marks)**

c. In a survey of 120 passengers, an airline found that 48 preferred ice cream with their meals, 78 preferred fruits and 66 preferred coffee. In addition, 36 preferred any given pair of these and 24 passengers preferred them all. If two passengers are selected at random from the survey sample of 120, what is the probability that
i) they both preferred only coffee with their meals
ii) they both preferred exactly two of three offerings. **(06 Marks)**

d. A student visits a sports club everyday from Monday to Friday after school hours and plays one of the three games: Cricket, Tennis and Football. In how many ways can he play each of the three games at least once during a week (from Monday to Friday)? **(04 Marks)**

2 a. Define Tautology. Prove that, for any propositions p, q, r the compound proposition $[(p \to q) \wedge (q \to r)] \to (p \to q)$ is a tautology using truth table. **(06 Marks)**

b. Prove the following logical equivalence without using truth table
$$(p \to q) \wedge [\neg q \wedge (r \vee \neg q)] \Leftrightarrow \neg(q \vee p)$$ **(04 Marks)**

c. Define the dual of a logical statement. Write down the dual of
$$[(p \vee T_0) \wedge (q \vee F_0)] \vee [(r \wedge s) \wedge T_0]$$ **(04 Marks)**

d. Simplify the following switching network. [Refer Fig.Q2(d)] **(06 Marks)**

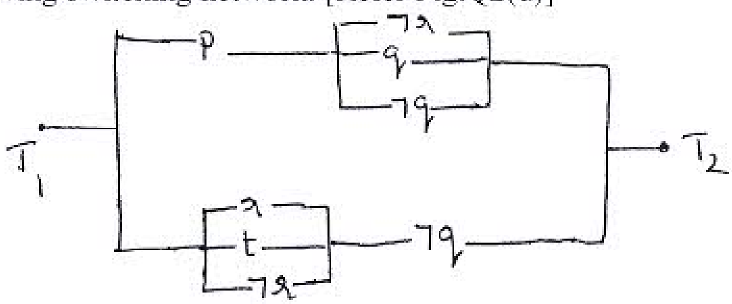

Fig.Q2(d)

3 a. If p(x): $x \geq 0$, q(x): $x^2 \geq 0$, r(x): $x^2 - 3x - 4 = 0$, s(x): $x^2 - 3 > 0$, find the truth values of the following:
i) $\exists x\, [p(x) \wedge q(x)]$ ii) $\forall x\, [p(x) \to q(x)]$ iii) $\forall x\, [q(x) \to s(x)]$
iv) $\forall x\, [r(x) \vee s(x)]$ v) $\exists x, [p(x) \wedge r(x)]$ vii) $\forall x, [r(x) \to p(x)]$ **(06 Marks)**

b. Negate and simplify each of the following:
i) $\forall x, [p(x) \wedge \neg q(x)]$ ii) $\exists x, [\{p(x) \vee q(x)\} \rightarrow r(x)]$ **(04 Marks)**

c. Establish the validity of the following argument:

$$\begin{array}{l} \forall x, [\, p(x) \vee q(x)\,] \\ \forall x, [\{\neg p(x) \wedge q(x)\} \rightarrow r(x)] \\ \hline \therefore \forall x, [\neg r(x) \rightarrow p(x)] \end{array}$$

(06 Marks)

d. Given R(x, y): x + y is even where the variables x and y represent integers write down the following in words:
i) $\forall x, \exists y\ p(x, y)$ ii) $\exists x, \forall y\ p(x, y)$ **(04 Marks)**

4 a. Prove that $4n < (n^2 - 7)$, for all integers $n \geq 6$. **(06 Marks)**

b. Obtain a recursive definition for the sequence $\{a_n\}$ in each of the following:
i) $a_n = 5n$ ii) $a_n = 2 - (-1)^n$ **(04 Marks)**

c. Prove that every positive integer $n \geq 24$ can be written as a sum of 5's and/or 7's. **(06 Marks)**

d. If $F_0, F_1, F_2, \ldots\ldots\ldots$ are Fibonacci numbers, prove that

$$\sum_{i=0}^{n} F_i^2 = F_n \times F_{n+1}$$, for all positive integers n. **(04 Marks)**

PART – B

5 a. Prove that a function $f : A \rightarrow B$ is invertible if and only if it is one-to-one and onto. **(06 Marks)**

b. Define Stirling number of second kind and evaluate S(8, 6). **(04 Marks)**

c. Let f, g, h be functions from z to z defined by $f(x) = x - 1$, $g(x) = 3x$,

$$h(x) = \begin{cases} 0 & \text{if } x \text{ is even} \\ 1 & \text{if } x \text{ is odd} \end{cases}$$

Determine $(f \circ (g \circ h))(x)$ and $((f \circ g) \circ h)(x)$ and verify that $f \circ (g \circ h) = (f \circ g) \circ h$. **(06 Marks)**

d. Show that if any (n + 1) numbers from 1 to 2n are chosen, then two of them will have their sum equal to (2n + 1). **(04 Marks)**

6 a. Let A = {1, 2, 3 ,4}, B = {w, x, y, z} and C = {5, 6, 7}. Also let R_1 be a relation from A to B, defined by $R_1 = \{(1, x),(2, x),(3, y),(3, z)\}$ and R_2 and R_3 be relations from B and C, defined by $R_2 = \{(w, 5),(x, 6)\}$, $R_3 = \{(w, 5),(w, 6)\}$. Find $R_1 \circ R_2$ and $R_1 \circ R_3$. **(06 Marks)**

b. Let A = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}. On this set define the relation R by $(x, y) \in R$ if and only if $(x - y)$ is a multiple of 5. Verify that R is an equivalence relation. **(04 Marks)**

c. Let (A, R_1) and (B, R_2) be points. On $A \times B$, define the relation R by $(a, b)R(x, y)$ if aR_1x and bR_2y. Prove that R is a partial order. **(06 Marks)**

d. Consider the Hasse diagram of a poset (A, R) given in Fig.Q6(c) below.

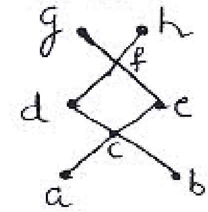

Fig.Q6(c)

If B = {c, d, e}

i) all upper bounds if B
ii) all lower bounds of B
iii) the least upper bound of B
iv) all greatest lower bound of B. **(04 Marks)**

7 a. Define subgroup of a group. Prove that H is a subgroup of a group G, if and only if, for all $a, b \in H$, $ab \in H$ and $\forall a \in H$, $a^{-1} \in H$. **(06 Marks)**

b. What is group homomorphism and group isomorphism? Give example for each. **(04 Marks)**

c. State and prove Lagrange's theorem. **(06 Marks)**

d. A binary symmetric channel has probability P = 0.05 of incorrect transmission. If the word C = 011011101 is transmitted, what is the probability that i) Single error occurs ii) a double error occurs iii) three errors occurs no two of them consecutive? **(04 Marks)**

8 a. Prove that the set Z with binary operations $\oplus$ and $\odot$ defined by $x \oplus y = x + y - 1$, $x \odot y = x + y - xy$ is a commutative ring with unity. **(06 Marks)**

b. Explain briefly the encoding and decoding of a message. **(04 Marks)**

c. The generator matrix for an encoding function, $E : Z_2^3 \to Z_2^6$ is given by

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

i) Find the code words assigned to 110 and 010
ii) Obtain the associated parity-check matrix.
iii) Hence decode the received words: 110110, 111101. **(06 Marks)**

d. Show that Z_5 is an integral domain. **(04 Marks)**

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