USN | | | | | | | | | | 10CS34

Third Semester B.E. Degree Examination, Dec.2014/Jan.2015
Discrete Mathematical Structures

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting atleast TWO questions from each part.***

PART – A

1 a. Simply the set expression $\overline{\overline{(A \cup B) \cap C} \cup \overline{B}}$ with justification. **(06 Marks)**

b. i) Use membership table to establish the set equality of : $\overline{(A \cap B) \cup (\overline{A} \cap C)} = (A \cap \overline{B}) \cup (\overline{A} \cap \overline{C})$. **(07 Marks)**

ii) If A = {1, 2, 3, 4, 5, 6, 7}, determine the number of subsets of A containing 3 elements, subsets of A containing 1, 2, and subsets of A with even number of elements.

c. The sample space of an experiment is S = {a, b, c, d, e, f, g, h}. If event A = {a, b, c} and event B = {a, c, e, g}, determine $P_r(A)$, $P_r(B)$, $P_r(A \cap B)$, $P_r(A \cup B)$, $P_r(\overline{A})$, $P_r(A \cap \overline{B})$ and $P_r(\overline{A} \cup B)$. **(07 Marks)**

2 a. Construct truth table for :

i) $[p \wedge (p \to q)] \to q$

ii) $[p \to q) \wedge (q \to r)] \to (p \to r)$. **(06 Marks)**

b. Simplify the switching network using the laws of logic. **(07 Marks)**

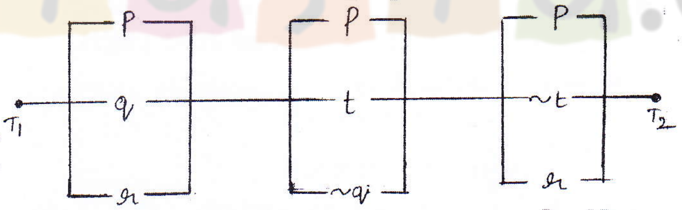

Fig. Q2(b)

c. Establish the following argument by the methods of proof by contradiction.

$P \to (q \wedge r)$

$r \to s$

$\sim (q \wedge s)$

$\therefore \sim p$. **(07 Marks)**

3 a. Negate and simplify each of the following :

i) $\exists x, [p(x) \vee q(x)]$

ii) $\forall x, [p(x) \wedge \sim q(x)]$

iii) $\forall x, [p(x) \to q(x)]$

iv) $\exists x, [(p(x) \vee q(x)) \to r(x)]$. **(06 Marks)**

b. Find whether the following argument is valid. No engineering student of first and second semester studies logic.

Anil is an engineering student who studies logic

∴ Anil is not in second semester **(07 Marks)**

c. Give : i) a direct proof ii) an indirect proof and iii) proof by contradiction for the following statement. "If m is an even integer, then m + 7 is odd". **(07 Marks)**

4 a. If $H_1 = 1$, $H_2 = 1 + \frac{1}{2}$ + , - - - - , $H_n = 1 + \frac{1}{2} + - - - - + 1/n$ are harmonic numbers, then prove that for all $n \in z^+$

$$\sum_{1=1}^{n} H_i = (n+1)H_n - n.$$ **(06 Marks)**

b. For all $n \in z^+$ prove that :

$$\sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{n+1}$$

$$\sum_{i=1}^{n} i(2^i) = 2 + (n-1)2^{n+1}.$$ **(07 Marks)**

c. i) If $A_1, A_2, - - - -, A_n \subseteq U$, then prove that :

$$\overline{A_1 \cap A_2 \cap - - - - - \cap A_n} = \overline{A_1} \cup \overline{A_2} \cup - - - - - \cup \overline{A_n}$$

ii) If $A, B_1, B_2, - - - - -, B_n \subseteq U$ then prove that $A \cap (B_1 \cup B_2 \cup - - - - \cup B_n) = (A \cap B_1) \cup (A \cap B_2) \cup - - - - \cup (A \cap B_n)$. **(07 Marks)**

PART – B

5 a. Find the number of ways of distributing 6 objects among 4 identical containers with some containers possibly empty. **(06 Marks)**

b. (i) Prove that the function $f : R \times R \to R$ defined by $f(a, b) = \lceil a + b \rceil$ is commutative but not associative

(ii) Prove that if 30 dictionaries in a library contains a total of 61,327 pages, then atleast one of the dictionary must have atleast 2045 pages. **(07 Marks)**

c. Let $f : R \to R$ be defined by

$$f(x) = \begin{cases} 3x - 5 & \text{for } x > 0 \\ -3x + 1 & \text{for } x \le 0 \end{cases}$$

then determine $f^{-1}(-1)$, $f^{-1}(3)$, $f^{-1}(6)$, $f^{-1}(-5, 5)$. **(07 Marks)**

6 a. Give a set A with $|A| = n$ and a relation R on A, let M denote the relation matrix for R then prove that :

i) R is symmetric if and only if $M = M^1$

ii) R is transitive if and only if $M.M = M^2 \le M$. **(06 Marks)**

b. Let $A = \{1, 2, 3, 4, 6, 8, 12\}$ and R be the partial ordering on A defined by $a R_b$ if a divides b then

i) Draw the Hasse diagram of the Poset (A, R)

ii) Determine the relation matrix for R

iii) Topologically sort the Poset (A, R) **(07 Marks)**

c. Let $A = \{1, 2, 3, 4, 5\} \times \{1, 2, 3, 4, 5\}$, and define R on A by $(x_1, y_1) R (x_2, y_2)$ if $x_1 + y_1 = x_2 + y_2$

i) Verify that R is an equivalence relation on A

ii) Determine the equivalence classes [(1, 3)], [(2, 4)] and [(1, 1)]

iii) Determine the partition of A induced by R. **(07 Marks)**

7 a. In a group S_6, let

$$\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 4 & 6 & 2 & 5 \end{pmatrix}$$

$$\beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 2 & 6 & 1 & 3 & 5 \end{pmatrix}$$

Determine $\alpha\beta$, α^3, β^4, $(\alpha\beta)^{-1}$. **(06 Marks)**

b. Define cyclic group and prove that every subgroup of a cyclic group is cyclic. **(07 Marks)**

c. Define the coding function $E: Z_2^3 \to Z_2^6$ by means of parity – check matrix $H = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$ and determine all code words. **(07 Marks)**

8 a. Let $E: Z_2^m \to Z_2^n$ be an encoding function given by a generator matrix G or the associated parity check matrix H then prove that $C = E:\left(Z_2^m\right)$ is a group code. **(06 Marks)**

b. i) Define subring and ideal

ii) If $A = \left\{\begin{bmatrix} a & o \\ b & c \end{bmatrix} a, b, c \in z\right\}$ be the subset of the ring $R = M_2(z)$ then prove that A is a subring but not ideal. **(07 Marks)**

c. i) Prove that Z_n is a field if and only if n is a prime.

ii) Prove that in Z_n, [a] is a unit if and only if gcd(a, n) = 1. **(07 Marks)**

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