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Fourth Semester B.E. Degree Examination, Dec.2014/Jan.2015
Engineering Mathematics – IV

Time: 3 hrs.

Max. Marks:100

Note: 1. Answer any FIVE full questions, selecting at least TWO questions from each part.
2. Use of statistical table is permitted.

PART – A

- 1** a. Employ Taylor's series method to find an approximate solution to find y at $x = 0.1$ given $\frac{dy}{dx} = x - y^2$, $y(0) = 1$ by considering upto fourth degree term. (06 Marks)
- b. Solve the following by Euler's modified method $\frac{dy}{dx} = \log(x + y)$, $y(0) = 2$ to find $y(0.4)$ by taking $h = 0.2$. (07 Marks)
- c. Given $\frac{dy}{dx} = x^2(Hy)$ and $y(1) = 1$, $y(1.1) = 1.233$, $y(1.2) = 1.548$, $y(1.3) = 1.979$. Evaluate $y(1.4)$ by Adams-Bashforth method. Apply corrector formula twice. (07 Marks)
- 2** a. Solve $\frac{dy}{dx} = 1 + xz$ and $\frac{dz}{dx} = -xy$ for $x = 0.3$ by applying Runge Kutta method given $y(0) = 0$ and $z(0) = 1$. Take $h = 0.3$. (06 Marks)
- b. Use Picard's method to obtain the second approximation to the solution of $\frac{d^2y}{dx^2} - x^3 \frac{dy}{dx} - x^3y = 0$ given $y(0) = 1$, $y'(0) = 0.5$. Also find $y(0.1)$. (07 Marks)
- c. Apply Milne's method to compute $y(0.4)$ given $y'' + xy' + y = 0$, $y(0) = 1$, $y'(0) = 0$, $y(0.1) = 0.995$, $y'(0.1) = -0.0995$, $y(0.2) = 0.9802$, $y'(0.2) = -0.196$, $y(0.3) = 0.956$ and $y'(0.3) = -0.2863$. (07 Marks)
- 3** a. Derive Cauchy-Riemann equation in Cartesian form. (06 Marks)
- b. Find an analytic function $f(z)$ whose real part is $\frac{\sin 2x}{\cosh 2y - \cos 2x}$ and hence find its imaginary part. (07 Marks)
- c. If $f(z)$ is a holomorphic function of z , then show that $\left\{ \frac{\partial}{\partial x} |f(z)| \right\}^2 + \left\{ \frac{\partial}{\partial y} |f(z)| \right\}^2 = |f'(z)|^2$. (07 Marks)
- 4** a. Discuss the transformation $w = z + \frac{1}{z}$. (06 Marks)
- b. Find the BLT which maps the points $z = 1, i, -1$ to $w = i, 0, -i$. Find image of $|z| < 1$. (07 Marks)
- c. Evaluate $\int_C \left\{ \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2(z-2)} \right\} dz$ where 'C' is circle $|z| = 3$. (07 Marks)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
 2. Any revealing of identification, appeal to evaluator and/or equations written eg. 42+8=50, will be treated as malpractice.

PART – B

- 5 a. Express $f(x) = x^4 + 3x^3 - x^2 + 5x - 2$ in terms of Legendre polynomials. (06 Marks)
- b. Obtain the solution of $x^2 y'' + xy' + (x^2 - x^2) y = 0$ in terms of $J_n(x)$ and $J_{-n}(x)$. (07 Marks)
- c. Derive Rodrigue's formula $P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} [(x^2 - 1)^n]$. (07 Marks)
- 6 a. State the axioms of probability. For any two events A and B, prove that,
 $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. (06 Marks)
- b. A box 'A' contains 2 white and 4 black balls. Another box 'B' contains 5 white and 7 black balls. A ball is transferred from the box A to the box B. Then a ball is drawn from the box B. Find the probability that it is white. (07 Marks)
- c. In a certain college 4% of the boys and 1% of girls are taller than 1.8m. Further more 60% of the students are girls. If a student is selected at random and is found to be taller than 1.8m, what is the probability that the student is a girl? (07 Marks)
- 7 a. The probability density of a continuous random variable is given by $p(x) = y_0 e^{-|x|}$, $-10 < x < \infty$. Find y_0 . Also find the mean. (06 Marks)
- b. Obtain the mean and variance of binomial distribution. (07 Marks)
- c. In a test on 2000 electric bulbs, it was found that the life of a particular make was normally distributed with an average life of 2040 hours and SD of 60 hours. Estimate the number of bulbs likely to burn for.
- More than 2150 hours.
 - Less than 1950 hours.
 - More than 1920 hours but less than 2160 hours.
- Given $A(1.5) = 0.4332$, $A(1.83) = 0.4664$, $A(2) = 0.4772$. (07 Marks)
- 8 a. In a city 'A' 20% of a random sample of 900 school boys had a certain slight physical defect. In another city B, 18.5% of a random sample of 1600 school boys had the same defect. Is the difference between the proportions is significant? Why? (06 Marks)
- b. A manufacturer claimed that atleast 95% of the equipment which he supplied to a factory conformed to specifications. An examination of a sample of 200 pieces of equipment revealed that 18 of them were faulty. Test his claim at a significance level of 1% and 5%. (07 Marks)
- c. A set of five similar coins is tossed 320 times and the result is

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|--------------|---|----|----|-----|----|----|
| No. of heads | 0 | 1 | 2 | 3 | 4 | 5 |
| Frequency | 6 | 27 | 72 | 112 | 71 | 32 |

Test the hypothesis that the data follow a binomial distribution $[x_{0.05}^2 = 11.07 \text{ for } 5df]$.

(07 Marks)

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