USN | | | | | | | | | | 10CS42

Fourth Semester B.E. Degree Examination, Dec.2014/Jan.2015
Graph Theory and Combinatorics

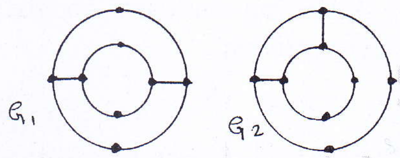
Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting atleast TWO questions from each part.***

PART – A

1 a. Define graph isomorphism and isomorphic graphs. Determine whether the following graphs are isomorphic or not: **(05 Marks)**

Fig.Q.1(a)



b. Define complement of a simple graph. Let G be a simple graph of order n. If the size of G is 56 and the size of $\overline{G}$ is 80. What is n? **(05 Marks)**

c. Let G = (V, E) be a connected undirected graph. What is the largest possible value for $|V|$ if $|E| = 19$ and $\deg(v) \geq 4$ for all $v \in V$? **(04 Marks)**

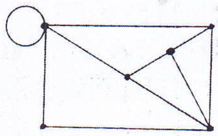
d. Write a note on "Konigsberg bridge problem and its solution". **(06 Marks)**

2 a. Define planar graph. Prove that the Peterson graph is nonplanar. **(05 Marks)**

b. Define Hamilton cycle. How many edge disjoint Hamilton cycles exist in the complete graph with seven vertices? Also draw the graph to show these Hamilton cycles. **(05 Marks)**

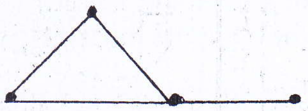
c. Define dual of a planar graph. Construct the dual of the planar graph given in Fig.Q.2(c). **(04 Marks)**

Fig.Q.2(c)



d. Define chromatic number and chromatic polynomial. Determine the chromatic polynomial for the graph show in Fig.Q.2(d). **(06 Marks)**

Fig.Q.2(d)

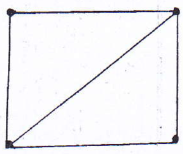


3 a. A class room contains 10 micro computer that are to be connected to a wall socket that has 2 outlets. Connections are made by using extension cords that have 2 outlets each. Find the least number of cords needed to get these computer set up for use. **(04 Marks)**

b. Apply merge sort to the list -1, 0, 2, -2, 3, 6, -3, 5, 1, 4. **(04 Marks)**

c. Find all the spanning trees of the graph shown in Fig.Q.3(c). Also find all the non isomorphic spanning trees. **(06 Marks)**

Fig.Q.3(c)



d. Obtain an optimal prefix code for the message MISSION SUCCESSFUL. Indicate the code for the message. **(06 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

4 a. State Krushkal's algorithm. Using Krushkal's algorithm find a minimal spanning tree for the weighted graph shown in Fig.Q.4(a). **(08 Marks)**

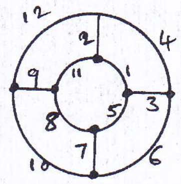


Fig.Q.4(a)

b. Apply Dijkstra's algorithm the diagram shown in Fig.Q.4(b) and determine the shortest distance from vertex a to each of the other vertices in the directed graph. **(06 Marks)**

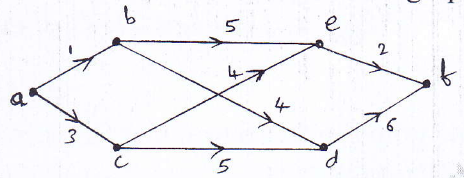


Fig.Q.4(b)

c. Define the following with one example for each: i) Cut set; ii) Edge connectivity; iii) Vertex connectivity. **(06 Marks)**

PART – B

5 a. A bit is either 0 or 1. A byte is a sequence of 8 bits. Find: i) The number of bytes; ii) The number of bytes that begin with 11 and end 11; iii) the number of bytes that begin with 11 and do not end with 11 and iv) the number of bytes that begin 11 or end with 11. **(06 Marks)**

b. How many arrangements of the letters in MISSISSIPPI have no consecutive S's? **(05 Marks)**

c. Find the coefficient of x° in the expansion of $3\left(x^2 - \frac{2}{x}\right)^{15}$. **(05 Marks)**

d. In how many ways can we distribute 7 apples and 6 oranges among 4 children so that each child gets at least 1 apple? **(04 Marks)**

6 a. How many integers between 1 and 300 (inclusive) are
i) Divisible by at least one of 5, 6, 8?
ii) Divisible by none of 5, 6, 8? **(06 Marks)**

b. Define derangement. Find the number of derangements of 1, 2, 3, 4. List all the derangements. **(06 Marks)**

c. Five teachers T_1, T_2, T_3, T_4, T_5 are to be made class teachers for five classes, C_1, C_2, C_3, C_4, C_5 one teacher for each class. T_1 and T_2 do not wish to become the class teachers for C_1 or C_2, T_3 and T_4 for C_4 or C_5, and T_5 for C_3 or C_4 or C_5. In how many ways can the teachers be assigned the work? **(08 Marks)**

7 a. Find the generating function for the sequence 8, 26, 54, 92…. **(06 Marks)**

b. Using generating function, find the number of i) non negative and ii) positive integer solutions of the equation $x_1 + x_2 + x_3 + x_4 = 25$. **(08 Marks)**

c. Define exponential generating functions using exponential generating function find the number of ways in which 5 of the letters in the word CALCULUS be arranged. **(06 Marks)**

8 a. The number of bacteria in a culture is 1000 (approximately) and this number increases 250% every two hours. Use a recurrence relation to determine the number of bacteria present after one day. **(05 Marks)**

b. Solve the recurrence relation $a_{n+2} - 4a_{n+1} + 3a_n = -200$, $n \geq 0$ and $a_0 = 3000$, $a_1 = 3300$. **(07 Marks)**

c. Find the generating function for the recurrence relation $a_{n+1} - a_n = n^2$, $n \geq 0$ and $a_0 = 1$. **(08 Marks)**

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