

Third Semester B.E. Degree Examination, December 2011
Engineering Mathematics – III

Time: 3 hrs.

Max. Marks:100

- Note: 1. Answer any FIVE full questions, selecting at least TWO questions from each part.**
2. Missing data will be suitably assumed.

PART – A

- 1 a. Obtain the Fourier series for the function $f(x) = \begin{cases} -\pi x & : 0 \leq x \leq 1 \\ \pi(2-x) & : 1 \leq x \leq 2 \end{cases}$ and deduce that

$$\frac{\pi^2}{8} = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}. \quad (07 \text{ Marks})$$

- b. Obtain the half range Fourier sine series for the function. (07 Marks)

$$f(x) = \begin{cases} 1/4 - x & ; 0 < x < 1/2 \\ x - 3/4 & ; 1/2 < x < 1 \end{cases}$$

- c. Compute the constant term and the first two harmonics in the Fourier series of $f(x)$ given by the following table. (06 Marks)

x	: 0	1	2	3	4	5
f(x)	: 4	8	15	7	6	2

- 2 a. Find the Fourier transform of $f(x) = \begin{cases} 1-x^2 & \text{for } |x| \leq 1 \\ 0 & \text{for } |x| > 1 \end{cases}$ and hence evaluate

$$\int_0^{\infty} \left(\frac{x \cos x - \sin x}{x^3} \right) \cos \frac{x}{2} dx. \quad (07 \text{ Marks})$$

- b. Find the Fourier cosine transform of $f(x) = \frac{1}{1+x^2}$. (07 Marks)

- c. Solve the integral equation $\int_0^{\infty} f(\theta) \cos \alpha \theta d\theta = \begin{cases} 1-\alpha & ; 0 \leq \alpha \leq 1 \\ 0 & ; \alpha > 1 \end{cases}$. Hence evaluate $\int_0^{\infty} \frac{\sin^2 t}{t^2} dt$. (06 Marks)

- 3 a. Solve two dimensional Laplace equation $u_{xx} + u_{yy} = 0$, by the method of separation of variables. (07 Marks)

- b. Solve the one dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$, $0 < x < \pi$ under the conditions :

i) $u(0,+) = 0, u(\pi, t) = 0$ ii) $u(x, 0) = u_0 \sin x$ where $u_0 = \text{constant} \neq 0$. (07 Marks)

- c. Obtain the D'Alembert's solution of one dimensional wave equation. (06 Marks)

- 4 a. Fit a curve of the form $y = ae^{bx}$ to the following data : (07 Marks)

x	: 77	100	185	239	285
y	: 2.4	3.4	7.0	11.1	19.6

- b. Using graphical method solve the L.P.P minimize $z = 20x_1 + 10x_2$ subject to the constraints $x_1 + 2x_2 \leq 40$; $3x_1 + x_2 \geq 0$; $4x_1 + 3x_2 \geq 60$; $x_1 \geq 0$; $x_2 \geq 0$. (06 Marks)

- c. Solve the following L.P.P maximize $z = 2x_1 + 3x_2 + x_3$, subject to the constraints $x_1 + 2x_2 + 5x_3 \leq 19$, $3x_1 + x_2 + 4x_3 \leq 25$, $x_1 \geq 0$, $x_2 \geq 0$, $x_3 \geq 0$ using simplex method. (07 Marks)

PART - B

- 5 a. Using the Regula - falsi method, find the root of the equation $x e^x = \cos x$ that lies between 0.4 and 0.6. Carry out four iterations. (07 Marks)
- b. Using relaxation method solve the equations :
 $10x - 2y - 3z = 205$; $-2x + 10y - 2z = 154$; $-2x - y + 10z = 120$. (07 Marks)
- c. Using the Rayleigh's power method, find the dominant eigen value and the corresponding eigen vector of the matrix. $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ starting with the initial vector $[1,1,1]^T$. (06 Marks)

- 6 a. From the following table, estimate the number of students who have obtained the marks between 40 and 45 : (07 Marks)
- | | | | | | |
|--------------------|---------|-------|-------|-------|-------|
| Marks | : 30-40 | 40-50 | 50-60 | 60-70 | 70-80 |
| Number of students | : 31 | 42 | 51 | 35 | 31 |
- b. Using Lagrange's formula, find the interpolating polynomial that approximate the function described by the following table : (07 Marks)

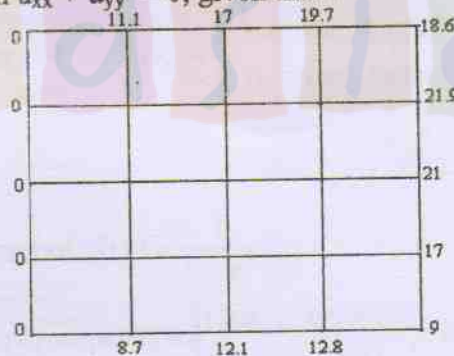
x	: 0	1	2	5	Hence find f(3)
f(x)	: 2	3	12	147	

- c. A curve is drawn to pass through the points given by the following table :

x	: 1	1.5	2	2.5	3	3.5	4
y	: 2	2.4	2.7	2.8	3	2.6	2.1

Using Weddle's rule, estimate the area bounded by the curve, the x - axis and the lines $x = 1$, $x = 4$. (06 Marks)

- 7 a. Solve the Laplace's equation $u_{xx} + u_{yy} = 0$, given that : (07 Marks)



- b. Solve $\frac{\partial^2 u}{\partial t^2} = 4 \frac{\partial^2 u}{\partial x^2}$ subject to $u(0, t) = 0$; $u(4, t) = 0$; $u(x, 0) = x(4 - x)$. Take $h = 1$, $k = 0.5$. (07 Marks)
- c. Solve the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ subject to the conditions $u(x, 0) = \sin \pi x$, $0 \leq x \leq 1$; $u(0, t) = u(1, t) = 0$ using Schmidt's method. Carry out computations for two levels, taking $h = 1/3$, $k = 1/36$. (06 Marks)

- 8 a. Find the Z - transform of : i) $(2n-1)^2$ ii) $\cos\left(\frac{n\pi}{2} + \pi/4\right)$ (07 Marks)

- b. Obtain the inverse Z - transform of $\frac{4z^2 - 2z}{z^3 - 5z^2 + 8z - 4}$. (07 Marks)

- c. Solve the difference equation $y_{n+2} + 6y_{n+1} + 9y_n = 2n$ with $y_0 = y_1 = 0$ using Z transforms. (06 Marks)

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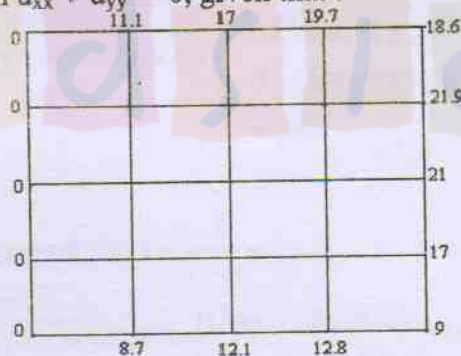
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