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Fourth Semester B.E. Degree Examination, December 2011
Graph Theory and Combinatorics

Time: 3 hrs.

Max. Marks:100

*Note: Answer FIVE full questions, selecting
 atleast TWO questions from each part.*

PART - A

- 1 a. Define regular graph. Draw a graph which has 10 – vertices and 15 edges and which should be a 3 – regular graph. (04 Marks)
- b. Define isomorphism of graphs. Determine whether the following graphs are isomorphic. (06 Marks)

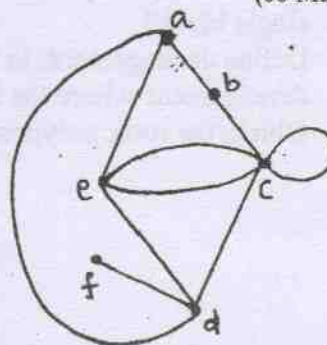
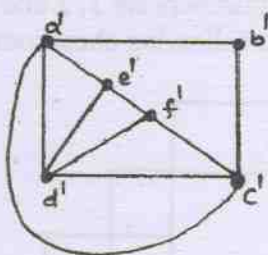
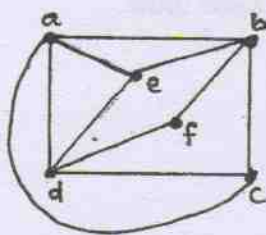


Fig. Q1(b)

Fig. Q2(b)

- c. If G is a graph with n – vertices and m – edges, let δ be the minimum and Δ be the maximal local degree, then prove that $\delta \leq \frac{2m}{n} \leq \Delta$. (05 Marks)
 - d. Give the comparison between Euler graph and Hamiltonian graph. (05 Marks)
- 2 a. Define complete bipartite graph. Prove that Kuratowski's second graph $K_{3,3}$ is nonplanar. (07 Marks)
 - b. Find the geometric dual of the following graph. (07 Marks)
 - c. Define chromatic number. Prove that a graph of order $n(\geq 2)$ consisting of a single circuit is 2 – chromatic if n is even, and 3 – chromatic if n is odd. (06 Marks)
- 3 a. Define a tree. Prove that a tree with n – vertices has exactly $(n - 1)$ edges. (07 Marks)
 - b. A tree has N_1 vertices of degree 1, N_2 vertices of degree 2, N_3 vertices of degree 3 and so on N_k vertices of degree k . prove that $N_1 = 2 + N_3 + 2N_4 + 3N_5 + \dots + (k - 2) N_k$. (06 Marks)
 - c. Obtain an optimal prefix code for the message "FALL OF THE WALL". (07 Marks)
- 4 a. Find a minimal spanning tree for the following graph using KRUSKAL'S algorithm. (07 Marks)

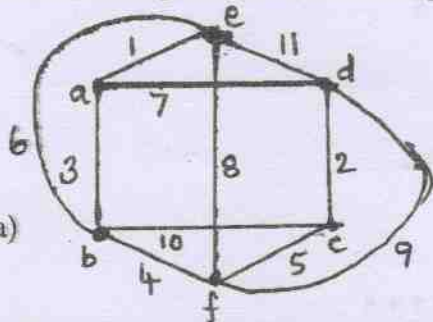


Fig. Q4(a)

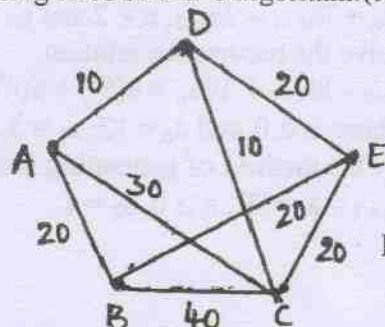


Fig. Q4(c)

- b. Define matching, complete matching, cutset of a graph. Give one example for each. (06 Marks)
- c. Explain the max – flow – min – cut theorem, apply this to network shown in Fig. Q4(c) to find the maximum flow possible between the vertices A and E. (07 Marks)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
 2. Any revealing of identification, appeal to evaluator and/or equations written eg, 42+8 = 50, will be treated as malpractice.

PART - B

- 5 a. Determine the number of 6 – digit integers (no leading 0's) in which
- No digit is repeated
 - No digit is repeated and it is even
 - No digit is repeated and it is divisible by 5. (08 Marks)
- b. Find the number of distinct terms in the expansion of $(w + x + y + z)^{12}$. (05 Marks)
- c. i) Show that $b_{n+1} = \frac{2(2n+1)}{(n+2)} b_n$, where b_n is the n^{th} Catalan number.
- ii) In how many ways can three semi-circles (can be of varying size) be drawn on a horizontal line so that the semi-circles do not intersect? Illustrate the various cases. (07 Marks)
- 6 a. In how many ways can 4a's, 3b's and 2c's be arranged so that all identical letters are not in a single block? (07 Marks)
- b. Define derangement. In how many ways can the integers 1, 2, 3, 4, 5 be deranged? List those derangement where the first three numbers are 1, 2 and 3 in some order. (07 Marks)
- c. Obtain the rook polynomial for the following chessboard. (06 Marks)

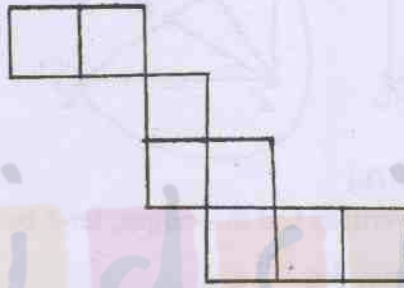


Fig. Q6(c)

- 7 a. In how many ways can two dozen identical robots be assigned to four assembly lines with
- Atleast three robots assigned to each line
 - Atleast three but not more than nine robots assigned to each line. (08 Marks)
- b. Show that the number of partitions of a positive integer n , where no summand appears more than twice equals the number of partitions of n , where no summand is divisible by 3. (06 Marks)
- c. i) Find the exponential generating function for the sequence $0!, 1!, 2!, 3! \dots$ and for a, a^3, a^5, a^7, \dots $a > 0$
- ii) Determine the sequence generated by the exponential generating function $3e^{3x}$. (06 Marks)
- 8 a. Solve the recurrence relation $2a_n = 7a_{n-1} - 3a_{n-2}$, $n \geq 2$ and $a_0 = 2, a_1 = 5$. (06 Marks)
- b. Solve the recurrence relation $a_{n+2} - 8a_{n+1} + 16a_n = 8(5^n) + 6(4^n)$ where, $n \geq 0$ and $a_0 = 12, a_1 = 5$. (08 Marks)
- c. By the method of generating function solve $a_{n+1} - a_n = 3^n, n \geq 0, a_0 = 1$. (06 Marks)
