Engineering Maths 2 - January 2014

TOTAL MARKS: 100
TOTAL TIME: 3 HOURS

(1) Question 1 is compulsory.
(2) Attempt any **four** from the remaining questions.
(3) Assume data wherever required.
(4) Figures to the right indicate full marks.

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1 (a) Choose the correct answer for the following: (4 marks)

(i) Suppose the equation to be solved is of the form, y=f(x, φ) then differentiating x we get equation of the form,

$$(a)\ \phi\left(x, p, \frac{dp}{dy}\right) = 0$$

$$(b)\ \phi\left(y, p, \frac{dp}{dx}\right) = 0$$

$$(c)\ \phi(x, yp) = 0$$

$$(d)\ \phi(x, y, 0) = 0$$

(ii) The general solution of the equation p^2-3p+2=0 is,
(a) (y+x-c)y+2x-c)
(b) (y-x-c)(y-2x-c)=0
(c) (-y-x-c)(y-2x-c)=0
(y-x-c)(y+x-c)=0
(iii) Clairaut's equation is of the form,
(a) x=py+f(p)
(b) y=p^2+f(p)
(c) y=px+f(p)
(d) None of these
(iv) Singular solution of y=px+2p^2 is,
(a) y^2+8y=0
(b) x^2-8y=0
(c) x^2+8y-c=0
(d) x^2+8y=0

1 (b) Solve p^2+2p cosh x+1=0. (4 marks)

1 (c) Find singular solution of p=sin(y-xp). (6 marks)

1 (d) Solve the equation y^2(y-xp)=x^4p^2 using substitution (6 marks)

$$X = \frac{1}{x} and Y = \frac{1}{y}$$

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2 (a) Choose the correct answer for the following: (4 marks)

(i) A second order linear differential equation has,
(a) two arbitary solution
(b) One arbitary solution
(c) no arbitary solution
(d) None of these
(ii) If 2, 4i and -4i are the roots of A.E of a homogeneous linear differential equation then its solution is,

$$(a)\ e^{x} + e^{x}(\cos 4x + \sin 4x)$$
$$(b)\ C_1 e^{2x} + C_2 \cos 4x + C_3 \sin 4x$$
$$(c)\ C_1 e^{2x} + C_2 e^{x} \cos 4x + C_3 e^{x} \sin 4x$$
$$(d)\ C_1 e^{2x} \cos 4x + C_2 e^{2x} \sin 4x$$

(iii) P.I. of $(D+1)^2$ $y=e^{-x+3}$

$$(a)\ \frac{x^2}{2}$$
$$(b)\ x^3 e^{x}$$
$$(c)\ \frac{x^3}{3} e^{-x=3}$$
$$(d)\ \frac{x^2}{2} e^{-x+3}$$

(iv) Particular integral of $f(D)y=e^{ax}$ V(x) is,

$$(a)\ \frac{e^{ax} V(x)}{f(D)}$$
$$(b)\ e^{ax} = \frac{1}{f(D)}[V(x)]$$
$$(c)\ e^{ax} \frac{1}{f(D+a)}[V(x)]$$
$$(d)\ \frac{1}{f(D+a)}[e^{ax} V(x)]$$

2 (b) (4 marks)

$$Solve\ \frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} + 3\frac{dy}{dx} - y = 0$$

2 (c) Solve y"-3y'+2y=2 sin x cos x (6 marks)

2 (d) Solve the system of equation, (6 marks)

$$\frac{dx}{dt} - 2y = \cos 2t,\ \frac{dy}{dt} + 2x = \sin 2t$$

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3 (a) Choose the correct answer for the following: (4 marks)

(i) In $x^2y'' + xy' - y = 0$ if $e^t = x$ then we get x^2y'' as,
(a) (D-1)y
(b) (D+1)y
(c) D(D+1)y
(d) None of these
(ii) In second order homogeneous differential equation $P_0(x)y''+P_1(x)y'+P_2(x)y=0$
x=a is a singular point if,
(a) $P_0(a)>0$
(b) $P_0(a)?0$
(c) $P_0(a)=0$
(d) $P_0(a)<0$
(iii) The general solution of

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0\ is,$$

$$(a)\ y = C_1x - C_2\frac{1}{x}$$

$$(b)\ C_1x + C_2\frac{1}{x}$$

$$(c)\ C_1x + C_2x$$

$$(d)\ C_1x - C_2x$$

(iv) Frobenius series solution of second order linear differential equation is of the form,

$$(a)\ x^m \sum_{r=0}^{\infty} a_r x^r$$

$$(b)\ \sum_{r=0}^{\infty} a_r x^r$$

$$(c)\ \sum_{r=a}^{\infty} a_r x^{m-r}$$

$$None\ of\ these$$

3 (b) Solve y"+a^2y=sec ax by the method of variation of parameters. (4 marks)

3 (c) (6 marks)

$$Solve\ x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = e^x$$

3 (d) Obtain the series solution of (6 marks)

$$\frac{dy}{dx} - 2xy = 0$$

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4 (a) Choose the correct answer for the following: (4 marks)

(i) PDE of az+b=a^2x+y is,

$$(a)\ \frac{\partial z}{\partial x}\cdot\frac{\partial z}{\partial y} = 1$$

$$(b)\frac{\partial z}{\partial x}\cdot\frac{\partial z}{\partial y} = 0$$

$$(c)\ \frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$$

$$(d)\ \frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 1$$

(ii) The solution of PDE Z_{xx}=2 y^2 is,

(a) z=x^2+xf(y)+ g(y)

(b) z=x^2y^2+xf(y)+g(y)

(c) $z=x^2y^2+f(x)+g(x)$
(d) $z=y^2+xf(y)+g(y)$
iii) The subsidiary equations of $(y^2+z^2)p+x(yq-z)=0$ are,

$$(a)\ \frac{dx}{p}=\frac{dy}{q}=\frac{dz}{R}$$
$$(b)\ \frac{dx}{y^2+z^2}=\frac{dy}{x}=\frac{dz}{xz}$$
$$(c)\ \frac{dx}{y^2+z^2}=\frac{dy}{xy}=\frac{dz}{xz}$$
$$(d)\ None\ of\ these$$

(iv) In the method of seperation of variable to solve $xz_n+z_t=0$ the assumed solution is of the form,
(a) X(x)Y(x)
(b) X(y)Y(y)
(c) X(t)Y(t)
(d) X(x)T(t)

4 (b) (4 marks)

$$Solve\ \frac{\partial^3 z}{\partial x^2 \partial y}=cos(2x+3y)$$

4 (c) Solve $xp-yq=y^2-x^2$ (6 marks)

4 (d) Solve $3u_x+2u_y=0$ by the seperation of variable method given that $u=4e^{-x}$ when y=0 (6 marks)

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5 (a) Choose the correct answer for the following: (4 marks)

$$\int_0^1\int_0^{x^2} e^{y/x}dydx = \_\_\_\_\_\_\_$$
$$(a)\ 1\ \ (b)\ -1/2\ \ (c)\ 1/2\ \ (d)\ None\ of\ these$$

(ii) The integral

$$\iint_R f(x,y)dxdy$$

by changing to polar form becomes,

$$(a)\ \iint_R \phi(r,\theta)drd\theta$$
$$(b)\ \iint_R f(r,\theta)drd\theta$$
$$(c)\ \iint_R f(r,\theta)rdrd\theta$$
$$(d)\ \iint_R \phi(r,\theta)rdrd\theta$$

(iii) For a real positive number n, the Gamma function ?(n)= _________

$$(a)\ \int_0^\infty x^{n-1}e^{-x}dx$$
$$(b)\ \int_0^1 x^{n-1}e^{-x}dx$$
$$(c)\ \int_0^x x^n e^{-x}dx$$
$$(d)\ \int_0^1 x^n e^{-x}dx$$

(iv) The Beta and Gamma functions relation for B(,n)= _______

$$(a)\ \frac{\Gamma(m)\Gamma(n)}{\Gamma(m-n)}$$
$$(b)\ \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$$
$$(c)\ \Gamma(m)\Gamma(n)$$
$$(d)\ \Gamma(mn)$$

5 (b) By changing the order of integration evaluate, (4 marks)

$$\int_0^a \int_{x/a}^{\sqrt{x/a}} (x^2+y^2)dydx,\ a>0$$

5 (c) (6 marks)

$$Evaluate \int_0^a \int_0^x \int_0^{x-y} e^{x+y+z} dzdydx$$

5 (d) Express the integral (6 marks)

$$\int_0^1 \frac{dx}{\sqrt{1-x^n}}$$

in terms of the Gamma function, Hence evaluate

$$\int_0^1 \frac{dx}{\sqrt{1-x^{2/3}}}$$

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6 (a) Choose the correct answer for the following: (4 marks)

(i) The scalar surface integral of

$$\overrightarrow{f}$$

over s, where s is a surface in a three-dimensional region R is given by,

$$\int \overrightarrow{f}.nds = \_\_\_\_\_\_\_$$

by using Gauss divergence theorem

$$(a) \iiint_v \nabla \cdot \overrightarrow{f} dV$$
$$(b) \iint_s \nabla \cdot \overrightarrow{t} dxdy$$
$$(c) \iiint_v \nabla \cdot \overrightarrow{F} dV$$
$$(d)\ None\ of\ these$$

(ii) If all the surface are closed in a region containing volume V then the following theorem is applicable.
(a) Stroke's theorem
(b) Green's theorem
(c) Gauss divergence theorem
(d) None of these
(iii) The value of

$$\int \{(2xy - x^2)dx + (x^2 + y^2)dx\}$$

by using Green's theorem is,
(a) Zeron (b) One (c) Two (d) Three
(iv)

$$\iint_s f.nds = \_\_\_\_\_\_\_$$

where f=xi+yj+2k and S is the surface of the sphere $x^2y^2+z^2=a^2$
(a) 4πa (b) $4\pi a^2$ (c) $4\pi a^3$ (d) 4π

6 (b) Find the work done by a force $f=(2y-x^2)i+ 6yzj-8xz^2k$ from the point (0, 0, 0) to the point (1, 1, 1) along the straight-line joining these points. (4 marks)

6 (c) If C is a simple closed curve in the xy-plane, prove by using Green's theorem that the integral (6 marks)

$$\int_C \frac{1}{2}(xdy - ydx)$$

represent the area A enclosed by . Hence evaluate

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

6 (d) Verify Stoke's theorem for (6 marks)

$$\vec{f} = (2x - y)i - yz^2j - y^2zk$$

for the upper half of the sphere $x^2+y^2+z^2=1$

7 (a) Choose the correct answer for the following: (4 marks)

(i) $L[t^n]=$ ________

$$(a)\ \frac{n}{s^{n+1}}$$
$$(b)\ \frac{n}{s^{n-1}}$$
$$(c)\ \frac{n!}{s^{n-1}}$$
$$(d)\ \frac{n!}{s^{n+1}}$$

(ii) $L[e^{-3t}]=$ _______

$$(a)\ \frac{3}{s-3}$$
$$(b)\ \frac{3}{s+3}$$
$$(c)\ \frac{1}{s+3}$$
$$(d)\ \frac{1}{s-3}$$

iii) L{f(t-a)H(t-a)} is equal to,

$$(a)\ \frac{3!}{(s+2)^4}$$
$$(b)\ \frac{3!}{(s-2)^4}$$
$$(c)\ \frac{3}{(s-2)^4}$$
$$(d)\ \frac{3}{(s-2)}$$

(iv) L{δ(t-1)}= _______

(a) e^{-s} (b) e^5 (c) e^{aS} (d) e^{-aS}

7 (b) Evaluate $L\{\sin^3 2t\}$ (6 marks)

7 (c) Find L{f(t)} given that (6 marks)

$$f(t)=\begin{cases}2 & 3>t>0\\ t & t>3\end{cases}$$

7 (d) Express (4 marks)

$$f(t)=\begin{cases}t^2 & 2>t>0\\ 4t & 4\geq t>2\\ 8 & t>4\end{cases}$$

in terms of unit step function and hence find their Laplace transform.

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8 (a) Choose the correct answer for the following: (4 marks)

(i) L^{-1} {cos at}= _______

$$(a)\ \frac{s}{s^2+a^2}$$
$$(b)\ \frac{s}{s^2-a^2}$$
$$(c)\ \frac{1}{s^2+a^2}$$
$$(d)\ \frac{1}{s^2-a^2}$$

(ii) L^{-1} $\{\overline{F}(s-a)\}$= ________

(a) $e^{t}f(t)$

(b) $e^{at}f(t)$

(c) $e^{-at}f(t)$

(d) None of these

$$L^{-1}\left\{\cot^{-1}\left(\frac{2}{s^2}\right)\right\}=\_\_\_\_\_\_$$
$$(a)\ \frac{\sin t}{t}$$
$$(b)\ \frac{\sinh at}{t}$$
$$(c)\ \frac{\sin at}{t}$$
$$(d)\ \frac{\sinh t}{t}$$

(iv) For the function f(t)=1, convolution theorem condition,

(a) Not satisfied
(b) Satisfied with some condition
(c) Satisfied
(d) None of these

8 (b) Find the inverse Laplace transform of (4 marks)

$$\frac{2s^2-6s+5}{(s-1)(s-2)(s-3)}$$

8 (c) Find (6 marks)

$$L^{-1}\left(\frac{s}{(s-1)(s^2+4)}\right)$$

using convolution theorem

8 (d) Solve differential equation y"(t) + y = F(t) where (6 marks)

$$F(t)=\begin{cases}0 & 1>t>0\\ 2 & t>1\end{cases}$$

Given that y(0)=0=y'(0)