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10MAT31

Third Semester B.E. Degree Examination, Dec.2013/Jan.2014

Engineering Mathematics – III

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. Find the Fourier series expansion of the function $f(x) = |x|$ in $(-\pi, \pi)$, hence deduce that $\frac{\pi^2}{8} = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}$. **(06 Marks)**

b. Obtain the half-range cosine series for the function, $f(x) = (x-1)^2$ in the interval $0 \le x \le 1$ and hence show that $\pi^2 = 8\left\{\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\right\}$ **(07 Marks)**

c. Compute the constant term and first two harmonics of the Fourier series of f(x) given by, **(07 Marks)**

x	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	2π
f(x)	1.0	1.4	1.9	1.7	1.5	1.2	1.0

2 a. Obtain the Fourier cosine transform of $f(x) = \frac{1}{1+x^2}$ **(06 Marks)**

b. Find the Fourier transform of $f(x) = \begin{cases} 1-x^2 & \text{for } |x| \le 1 \\ 0 & \text{for } |x| > 1 \end{cases}$ and evaluate $\int_0^{\infty} \frac{x\cos x - \sin x}{x^3} dx$. **(07 Marks)**

c. Find the inverse Fourier sine transform of $\frac{s}{1+s^2}$. **(07 Marks)**

3 a. Obtain the various possible solutions of two dimensional Laplace's equation, $u_{xx} + u_{yy} = 0$ by the method of separation of variables. **(07 Marks)**

b. Solve the one-dimensional wave equation, $C^2 \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}$, $0 < x < l$ under the following conditions (i) $u(0, t) = u(l, t) = 0$ (ii) $u(x, 0) = \frac{u_0 x}{l}$ where u_0 is constant (iii) $\frac{\partial u}{\partial t}(x, 0) = 0$. **(07 Marks)**

c. Obtain the D'Almbert's solution of the wave equation $u_{tt} = C^2 u_{xx}$ subject to the conditions $u(x, 0) = f(x)$ and $\frac{\partial u}{\partial t}(x, 0) = 0$. **(06 Marks)**

4 a. Find the best values of a, b, c, if the equation $y = a + bx + cx^2$ is to fit most closely to the following observations. **(07 Marks)**

x	1	2	3	4	5
y	10	12	13	16	19

b. Solve the following by graphical method to maximize $z = 50x + 60y$ subject to the constraints, $2x + 3y \le 1500$, $3x + 2y \le 1500$, $0 \le x \le 400$ and $0 \le y \le 400$. **(06 Marks)**

c. By using Simplex method, maximize $P = 4x_1 - 2x_2 - x_3$ subject to the constraints, $x_1 + x_2 + x_3 \le 3$, $2x_1 + 2x_2 + x_3 \le 4$, $x_1 - x_2 \le 0$, $x_1 \ge 0$ and $x_2 \ge 0$. **(07 Marks)**

PART – B

5 a. Using Newton-Raphson method, find a real root of $x\sin x + \cos x = 0$ nearer to π, carryout three iterations upto 4-decimals places. **(07 Marks)**

b. Find the largest eigen value and the corresponding eigen vector of the matrix,

$$\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

By using the power method by taking the initial vector as $[1\ 1\ 1]^T$ carryout 5-iterations. **(07 Marks)**

c. Solve the following system of equations by Relaxation method:
$12x + y + z = 31$; $2x + 8y - z = 24$; $3x + 4y + 10z = 58$ **(06 Marks)**

6 a. A survey conducted in a slum locality reveals the following information as classified below.

Income per day in Rupees 'x'	Under 10	10 – 20	20 – 30	30 – 40	40 – 50
Numbers of persons 'y'	20	45	115	210	115

Estimate the probable number of persons in the income group 20 to 25. **(07 Marks)**

b. Determine f(x) as a polynomials in x for the data given below by using the Newton's divided difference formula. **(07 Marks)**

x	2	4	5	6	8	10
f(x)	10	96	196	350	868	1746

c. Evaluate $\int_0^1 \frac{x}{1+x^2}dx$ by using Simpson's $\left(\frac{1}{3}\right)^{rd}$ rule by taking 6 – equal strips and hence deduce an approximate value of $\log_e 2$. **(06 Marks)**

7 a. Solve the wave equation, $\frac{\partial^2 u}{\partial t^2} = 4\frac{\partial^2 u}{\partial x^2}$, subject to u(0, t) = 0, u(4, t) = 0, $u_t(x, 0) = 0$ and u(x, 0) = x(4 – x) by taking h = 1, K = 0.5 upto 4-steps. **(07 Marks)**

b. Solve numerically the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ subject to the conditions, u(0, t) = 0 = u(1, t), $t \geq 0$ and u(x, 0) = sin πx, $0 \leq x \leq 1$, carryout the computation for two levels taking $h = \frac{1}{3}$ and $K = \frac{1}{36}$. **(07 Marks)**

c. Solve $u_{xx} + u_{yy} = 0$ in the following square region with the boundary conditions as indicated in the Fig. Q7 (c). **(06 Marks)**

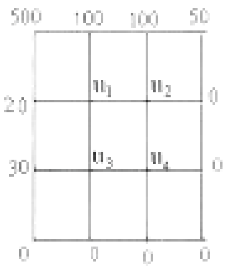

Fig. Q7 (c)

8 a. Find the z-transform of, (i) sinh nθ (ii) cosh nθ (iii) n^2 **(07 Marks)**

b. Find the inverse z-transform of, $\frac{2z^2 + 3z}{(z+2)(z-4)}$. **(06 Marks)**

c. Solve the difference equation, $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$ with $y_0 = y_1 = 0$ by using z-transform. **(07 Marks)**

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