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10MAT11

First Semester B.E. Degree Examination, January 2011

Engineering Mathematics – I

Time: 3 hrs. Max. Marks:100

Note: 1. ***Answer any FIVE full questions, choosing at least two from each part.***
2. ***Answer all objective type questions only on OMR sheet page 5 of the answer booklet.***
3. ***Answer to objective type questions on sheets other than OMR will not be valued.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

PART – A

1 a. Choose the correct answer :

i) If f(x) is continuous in [a, b], differentiable in (a, b) and f(a)= f(b), then there exists ____ $C \in (a, b)$ such that $f'(c) = 0$.

A) unique B) infinite C) at least one D) no such

ii) The Maclaurin's series of f(x) = k(constant) is,

A) f(x) = k B) f(x) = 0 C) does not exist D) f(x) = k!

iii) The n^{th} derivative of $\dfrac{1}{(x+2)^3}$ is

A) $\dfrac{(-1)^n(n+2)!}{2!(x+2)^{n+3}}$ B) $\dfrac{1}{(x+2)^{n+3}}$ C) ZERO D) None of these.

iv) The 12^{th} derivative of $y = e^{\sqrt{2}x}\sin\sqrt{2}x$ is

A) (64)y B) -4096y C) (32)y D) None of these.

(04 Marks)

b. If x = tan(log y), prove that $(1+x^2)y_{n+1} + (2nx - 1)y_n + n(n-1)y_{n-1} = 0$ **(06 Marks)**

c. Expand log(sec x) by using the Maclaurin's series expansion up to the term containing x^4. **(05 Marks)**

d. State and prove the Lagrange's mean value theorem. **(05 Marks)**

2 a. Choose the correct answer :

i) Which statement is true?

A) $\dfrac{0}{0}, \dfrac{\infty}{\infty}, \infty - \infty, 0\times\infty$ are not indeterminate B) 0^0, ∞^0 are not indeterminate

C) 1^∞ is not indeterminate D) None of these.

ii) The angle between $r = a\sin\theta$ and $r = b\cos\theta$, is

A) π/2 B) π C) - π/2 D) None of these.

iii) The radius of a curvature in the polar form is,

A) $\dfrac{[r^2+r_1^2]^{3/2}}{r^2+2r_1^2-rr_2}$ B) $\dfrac{[r_1^2+r^2]^{3/2}}{r_1^2+2r^2-rr_2}$ C) $\dfrac{[r^2+r_1^2]^{3/2}}{r^2+2r_1r_2-rr_2}$ D) None of these.

iv) $\lim\limits_{x\to 0}\dfrac{2^x-3^x}{5^x-6^x}$ is ,

A) $\dfrac{\log(2/3)}{\log(5/6)}$ B) $\log\left[\dfrac{2}{3}-\dfrac{5}{6}\right]$ C) $\log\left[\dfrac{2/3}{5/6}\right]$ D) None of these.

(04 Marks)

b. Evaluate : i) $\lim\limits_{x\to 0}\dfrac{\sin x\sin^{-1}x}{x^2}$ ii) $\lim\limits_{x\to 0}\left(\dfrac{2^x+3^x+4^x}{3}\right)^{1/x}$ **(06 Marks)**

c. Derive an expression for the radius of curvature in the pedal form. **(05 Marks)**

d. Find the radius of curvature of $a^2y = x^3 - a^3$ at the point where the curve cuts x-axis. **(05 Marks)**

3 a. Choose the correct answer :

i) If $u = ax^2 + by^2 + abxy$, then $\dfrac{\partial^3 u}{\partial x^2 \partial y}$ is

A) Zero B) a + b + ab C) ab D) None of these.

ii) The Taylor's series of f(x, y) = xy at (1, 1) is

A) $1 + [(x-1)+(y-1)]$ B) $1 + [(x-1)+(y-1)] + [(x-1)(y-1)]$

C) $(x-1)(y-1)$ D) None of these.

iii) The Jacobian of transformation from the Cartesian to polar coordinate system is,

A) r^2 B) $r^2\cos\theta$ C) $r^2\sin\theta$ D) None of these.

iv) If $u = f(x, y)$, $x = \phi(t)$, $y = \psi(t)$, then du/dt is,

A) $\dfrac{du}{dx}\dfrac{dx}{dt} + \dfrac{du}{dy}\dfrac{dy}{dt}$ B) $\dfrac{dx}{dt} + \dfrac{dy}{dt}$ C) $\dfrac{\partial u}{\partial x}\dfrac{dx}{dt} + \dfrac{\partial u}{\partial y}\dfrac{dy}{dt}$ D) None of these.

(04 Marks)

b. If $\sin u = \dfrac{x^2 y^2}{x+y}$, show that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = 3\tan u$ **(06 Marks)**

c. If $u = \dfrac{xy}{z}$, $v = \dfrac{yz}{x}$ and $w = \dfrac{xz}{y}$, find $J = \dfrac{\partial(u,v,w)}{\partial(x,y,z)}$. **(05 Marks)**

d. If the H.P. required by the steamer varies as the cube of the velocity and the square of the length, find the percentage change in H.P. for 3% and 4% increase in velocity and length respectively. **(05 Marks)**

4 a. Choose the correct answer :

i) The gradient, divergence, curl are respectively

A) scalar, scalar, vector B) vector, scalar, vector

C) scalar, vector, vector D) vector, vector, scalar

ii) $\vec{V} = y^2 z\,\vec{i} + z^2 x\,\vec{j} + x^2 y\,\vec{k}$ is

A) constant vector B) solenoidal vector C) scalar D) None of these.

iii) Curl grad f is,

A) grad curl f B) curl grad f + grad curl f C) zero D) does not exist.

iv) If the curvilinear system is spherical polar coordinate system then the radius vector R is

A) $r\sin\theta\cos\phi\,\vec{i} + r\sin\theta\sin\phi\,\vec{j} + r\cos\theta\,\vec{k}$ B) $r\sin\theta\,\vec{i} + r\cos\theta\,\vec{j} + r\vec{k}$

C) $\vec{i} + \vec{j} + \vec{k}$ D) None of these. **(04 Marks)**

b. If $\phi = x^2 + y^2 + z^2$ and $\vec{F} = x^2\vec{i} + y^2\vec{j} + z^2\vec{k}$, then find grad$\phi$, div$\vec{F}$, curl$\vec{F}$. **(06 Marks)**

c. Prove that div Curl F = $\nabla\cdot\nabla\times F = 0$. **(05 Marks)**

d. Prove that the cylindrical coordinate system is orthogonal. **(05 Marks)**

PART – B

5 a. Choose the correct answer :

i) The value of $\int_0^{\pi} \sin^5 x \cos^6 x\, dx$ is

A) $\dfrac{5\times3\times1}{11\times9\times7}$ B) $\dfrac{4\times2}{11\times9}\,\dfrac{\pi}{2}$ C) $\dfrac{2\times4\times2}{11\times9\times7}$ D) None of these.

ii) $x^2 + y^2 = x^2 y^2$ is symmetric about

A) x-axis B) y-axis C) the line y = x D) All of these

iii) Surface area of a solid of revolution of the curve y = f(x), if rotated about x-axis, is:

A) $\int_{x=a}^{b} 2\pi y\, dx$ B) $\int_{x=a}^{b} 2\pi x\, dy$ C) $\int_{x=a}^{b} 2\pi y\, ds$ D) $\int_{x=a}^{b} 2\pi x\, ds$

iv) Asymptote to the curve $y^2(a - x) = x^3$ is

A) $y = 0$ B) $x = 0$ C) $x = a$ D) None of these.

(04 Marks)

b. Evaluate $\int_0^1 \frac{x^\alpha - 1}{\log x} dx$, $\alpha \geq 0$. **(06 Marks)**

c. Derive the reduction formula for $\int_0^{\pi/2} \sin^n x \, dx$. **(05 Marks)**

d. Compute the perimeter of the cardiod $r = a(1 + \cos\theta)$. **(05 Marks)**

6 a. Choose the correct answer :

i) For the differential equation $\left(\frac{d^3y}{dx^3}\right)^2 + \left(\frac{d^2y}{dx^2}\right)^6 + y = x^4$, the order and degree respectively are

A) 2, 6 B) 3, 2 C) 2, 4 D) None of these.

ii) $\frac{dy}{dx} + \frac{y}{x} = 0$ is

A) Variable separable and homogeneous B) Linear

C) Homogeneous and exact D) All of these.

iii) $ydx - xdy = 0$ can be reduced to exact, if divided by

A) $x^2 + y^2$ B) y^2 C) xy D) All of these.

iv) Orthogonal trajectory of $y^2 = 4a(x + a)$ is

A) $x^2 = 4a(y + a)$ B) $x^2 + y^2 = a^2$ C) Self orthogonal D) None of these.

(04 Marks)

b. Solve: $(1 + y^2)dx + (x - e^{-\tan^{-1}y})dy = 0$ **(06 Marks)**

c. Solve: $(y^2e^{xy^2} + 4x^3)dx + (2xye^{xy^2} - 3y^2)dy = 0$ **(05 Marks)**

d. Find the orthogonal trajectory of the cardiods $r = a(1 - \cos\theta)$, using the differential equation method. **(05 Marks)**

7 a. Choose the correct answer :

i) Which of the following is not an elementary transformation?

A) Adding two rows B) Adding two columns

C) Multiplying a row by a non-zero number D) Squaring all the elements of the matrix.

ii) Rank of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$ is

A) 3 B) 1 C) 2 D) None of these.

iii) The solution of the simultaneous equations $x + y = 0$, $x - 2y = 0$ is

A) only trivial B) only unique C) unique and trivial D) None of these.

iv) Which of the following is in the normal form?

A) $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ B) $B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ C) $C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ D) All of these.

(04 Marks)

b. Find the rank of the matrix $\begin{bmatrix} 91 & 92 & 93 & 94 & 95 \\ 92 & 93 & 94 & 95 & 96 \\ 93 & 94 & 95 & 96 & 97 \\ 94 & 95 & 96 & 97 & 98 \\ 95 & 96 & 97 & 98 & 99 \end{bmatrix}$. **(06 Marks)**

c. For what values of λ and μ, the following simultaneous equations have i) No solution ii) a unique solution iii) an infinite number of solutions?
$x + y + z = 6$; $x + 2y + 3z = 10$; $x + 2y + \lambda z = \mu$. **(05 Marks)**

d. Solve, using the Gauss-Jordan method.
$x + y + z = 9$; $x - 2y + 3z = 8$; $2x + y - z = 3$. **(05 Marks)**

8 a. Choose the correct answer :

i) The eigen values of the matrix A exist, if
A) A is a square matrix B) A is singular matrix
C) A is any matrix D) A is a null matrix.

ii) A square matrix A of order 'n' is similar to a square matrix B of the order 'n' if
A) $A = P^{-1}BP$ B) AB = Null matrix C) AB = Unit matrix D) None of these.

iii) Which of these is in quadratic form?
A) $x^2 + y^2 + z^2 - 2xy + yz - zx$ B) $x^3 + y^3 + z^2$
C) $(x - y + z)^2$ D) None of these.

iv) Quadratic form (X′AX) is positive definite, if
A) All the eigen values of A are > 0 B) At least one eigen value of A is > 0
C) All eigen values ≥ 0 and at least one eigen value = 0 D) No such condition. **(04 Marks)**

b. Find the eigen values and eigen vector corresponding to the largest eigen value of the matrix
$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ **(06 Marks)**

c. If $P = \begin{bmatrix} -1 & 1 & 1 \\ 0 & -1 & 2 \\ 1 & 1 & 1 \end{bmatrix}$ is a modal matrix of the matrix A in Q.No.8(b), and inverse of P is
$P^{-1} = \begin{bmatrix} -3 & 0 & 3 \\ 2 & -2 & 2 \\ 1 & 2 & 1 \end{bmatrix}$, then transform A in to diagonal form and hence find A^4. **(05 Marks)**

d. Find the nature of the quadratic forms for which corresponding eigen values of the corresponding matrices are given as

Matrix	Eigen values
A	2, 3, 4
B	–3, –4, –5
C	0, 3, 6
D	0, –3, –4
E	–2, 3, 4

(05 Marks)

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