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MAT21

Second Semester B.E. Degree Examination, December 2010
Engineering Mathematics - II

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting at least ONE question from each part.***

PART – A

1 a. Find the radius of curvature at any point θ of the curve $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$. **(06 Marks)**

b. Show that the constant C of Cauchy's mean value theorem for the functions sinx and cosx in (a, b) is the arithmetic mean between a and b $(0 < a < b)$. **(07 Marks)**

c. Expand $\log(1 + \sin x)$ in powers of x by Maclaurin's theorem upto the term containing x^4. **(07 Marks)**

2 a. Evaluate: $\lim_{x \to 0}\left(\frac{\sin x}{x}\right)^{\frac{1}{x^2}}$ **(06 Marks)**

b. Expand $\tan^{-1}(y/x)$ about the point (1, 1) upto second degree terms. **(07 Marks)**

c. Examine the function $f(x, y) = x^4 + y^4 - 2(x - y)^2$ for extreme values. **(07 Marks)**

PART – B

3 a. Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz\, dz\, dy\, dx$. **(06 Marks)**

b. Change the order of integration and evaluate $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} dy\, dx$ **(07 Marks)**

c. Show that $\int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin\theta}} \times \int_0^{\pi/2} \sqrt{\sin\theta}\, d\theta = \pi$ **(07 Marks)**

4 a. A particle moves so that its position vector is given by $\vec{r} = \cos wt\,\hat{i} + \sin wt\,\hat{j}$, where w is a constant. Show that the velocity $\vec{v}$ of the particle is perpendicular to $\vec{r}$ and $\vec{r} \times \vec{v}$ and is a constant vector. **(06 Marks)**

b. If $\vec{f} = (x + y + az)\hat{i} + (bx + 2y - z)\hat{j} + (x + cy + 2z)\hat{k}$, find a, b, c such that curl $\vec{f} = 0$. Then find ϕ such that $\vec{f} = \nabla\phi$. **(07 Marks)**

c. If $\vec{f} = xy^2\hat{i} + yz^2\hat{j} + zx^2\hat{k}$ and S is the part of the paraboloid $z = a^2 - x^2 - y^2$ above the xy-plane, show by using Stoke's theorem that $\int_S (\text{curl}\,\vec{f}).\hat{n}\,ds = 0$ **(07 Marks)**

PART – C

5 a. Solve : $\frac{d^2y}{dx^2}+9y=\cos 2x\cos x$ **(06 Marks)**

b. Solve : $\frac{d^2y}{dx^2}+3\frac{dy}{dx}+2y=12x^2$ **(07 Marks)**

c. Using the method of variation of parameters, find the solution of $\frac{d^2y}{dx^2}+y=\tan x$. **(07 Marks)**

6 a. Using the method of undetermined coefficients, solve the differential equation $\frac{d^2y}{dx^2}-3\frac{dy}{dx}+2y=x^2+e^x$ **(06 Marks)**

b. Obtain the solution of differential equation

$(2x+1)^2\frac{d^2y}{dx^2}-2(2x+1)\frac{dy}{dx}-12y=6x+5$ **(07 Marks)**

c. Solve : $\frac{d^2y}{dx^2}+4\frac{dy}{dx}+5y=0$. Given that $y=2$ and $\frac{dy}{dx}=\frac{d^2y}{dx^2}$, when $x=0$ **(07 Marks)**

PART – D

7 a. Find the Laplace transforms of the following :

i) $t^2\sin at$ ii) $\frac{1-e^{-at}}{t}$ **(06 Marks)**

b. Given $f(t)=\begin{cases} E & 0<t<a/2 \\ -E & a/2<t<a \end{cases}$, where $f(t+a)=f(t)$. Show that $L\{f(t)\}=\frac{E}{S}\tanh\left(\frac{as}{4}\right)$. **(07 Marks)**

c. Given $f(t)=\begin{cases} \sin t, & 0<t<\pi/2 \\ \cos t, & t>\pi/2 \end{cases}$

Express f(t) in terms of the unit step function. Hence find the Laplace transform. **(07 Marks)**

8 a. Find the inverse Laplace transform of $\frac{4s+5}{(s+1)^2(s+2)}$. **(06 Marks)**

b. By employing convolution theorem, evaluate $L^{-1}\left[\frac{1}{(s-1)(s^2+1)}\right]$. **(07 Marks)**

c. Using the Laplace transform method,

solve $\frac{d^2y}{dx^2}+6\frac{dy}{dx}+9y=12t^2e^{-3t}$, with $y(0)=0=y'(0)$ **(07 Marks)**

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