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Second Semester B.E. Degree Examination, December 2011
Engineering Mathematics – II

Time: 3 hrs.

Max. Marks:100

- Note:** 1. Answer any FIVE full questions, choosing at least two from each part.
 2. Answer all objective type questions only on OMR sheet page 5 of the answer booklet.
 3. Answer to objective type questions on sheets other than OMR will not be valued.

PART – A

- 1 a. Choose your answers for the following : (04 Marks)
- i) The general solution of the equation $yp^2 + (x-y)p - x = 0$ is
- A) $(x-y-c)(x^2+y^2-c) = 0$ B) $(y-x-c)(x^2-y^2-c) = 0$
 C) $(y-x-c)(y^2-x^2-c) = 0$ D) $(y-x-c)(x^2+y^2-c) = 0$
- ii) The given differential equation is solvable for x, if it is possible to express x in terms of,
- A) x and y B) x and p C) y and p D) None of these
- iii) The singular solution of the equation $y = px + \frac{a}{p}$ is
- A) $y^2 = 4ax$ B) $x^2 = 4ay$ C) $x^2 = y$ D) $y^2 = x$
- iv) The general solution of Clairaut's equation is,
- A) $y = cx + f(c)$ B) $x = cy + f(c)$ C) $y = cx - f(c)$ D) None of these
- b. Solve : $p(p+y) = x(x+y)$. (04 Marks)
- c. Obtain the general solution and the singular solution of the equation, $y = 2px + p^2y$. (06 Marks)
- d. Obtain the general and singular solution of Clairaut's equation, $xp^3 - yp^2 + 1 = 0$. (06 Marks)
- 2 a. Choose your answers for the following : (04 Marks)
- i) The particular integral of $(D^2 + a^2)y = \sin ax$ is
- A) $-\frac{x}{2a} \cos ax$ B) $\frac{x}{2a} \cos ax$ C) $-\frac{ax}{2} \cos ax$ D) $\frac{ax}{2} \cos ax$
- ii) The solution of the differential equation $y'' + y = 0$ satisfying the conditions $y(0) = 1$ and $y\left(\frac{\pi}{2}\right) = 2$ is
- A) $y = \cos x - 2 \sin x$ B) $y = 2 \sin x - \cos x$
 C) $y = \cos x + 2 \sin x$ D) $y = C_1 \cos x + C_2 \sin x$
- iii) P.I of $(D+1)^2 y = xe^{-x}$ is,
- A) $\frac{x}{6} e^{-x}$ B) $\frac{x^3}{6} e^{-x}$ C) $-\frac{x^3}{6} e^{-x}$ D) $\frac{x^2}{2} e^{-x}$
- iv) P.I of $(D^2 + D)y = x^2 + 2x + 4$ is
- A) $\frac{x^2}{3} + 4x$ B) $\frac{x^3}{3} + 4$ C) $\frac{x^3}{3} + 4x$ D) $\frac{x^3}{3} + 4x^2$
- b. Solve : $(D-2)^2 y = 8(e^{2x} + \sin 2x)$ (04 Marks)
- c. Solve : $y'' - 2y' + y = x \cos x$ (06 Marks)
- d. Solve : $\frac{dx}{dt} - 7x + y = 0, \frac{dy}{dt} - 2x - 5y = 0$. (06 Marks)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
 2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

3 a. Choose your answers for the following :

(04 Marks)

- i) The complementary function of the equation $x^2y'' - xy' + y = \log x$ is
A) $y = (C_1 + C_2x)e^x$ B) $y = (C_1 + C_2 \log x)x$
C) $y = (C_1 + C_2x)x$ D) $y = C_1e^x + C_2e^{-x}$
- ii) The homogeneous linear differential equation whose auxillary equation has roots 1, -1 is
A) $x^2y_2 - xy_1 + y = 0$ B) $x^2y_2 - xy_1 - y = 0$
C) $y'' - y = 0$ D) $x^2y_2 + xy_1 - y = 0$
- iii) To transform $xy'' + y' = \frac{1}{x}$ into a linear differential equation with constant coefficients put $x = \dots\dots\dots$
A) e^t B) e^{-t} C) $\log t$ D) None of these
- iv) The solution of $x^2y'' + xy' = 0$ is
A) $y = C_1 \cos x + C_2 \sin x$ B) $y = C_1e^x + C_2e^{-x}$
C) $y = a \log x + b$ D) $y = C_1 + 6x^3$

b. Solve $y'' - 6y' + 9y = \frac{e^{3x}}{x^2}$ by the method of variation of parameters. (04 Marks)

c. Solve : $(1+x^2)\frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = 2\sin[\log(1+x)]$. (06 Marks)

d. Solve by Frobenius method the equation: $4x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$. (06 Marks)

4 a. Choose your answers for the following :

(04 Marks)

- i) The solution of $\frac{\partial^2 z}{\partial y^2} = \sin(xy)$ is
A) $z = -x^2 \sin(xy) + yf(x) + g(x)$ B) $z = -x^2 \cos(xy) - yf(x) + g(x)$
C) $z = -\frac{\sin(xy)}{x^2} + yf(x) + g(x)$ D) None of these

- ii) A solution of $(y-z)p + (z-x)q = x-y$ is
A) $x^2 + y^2 + z^2 = f(x-y-z)$ B) $x^2 + y^2 + z^2 = f(x+y+z)$
C) $x^2 - y^2 - z^2 = f(x+y+z)$ D) $x^2 + y^2 - z^2 = f(x+y+z)$

- iii) The partial differential equation obtained from $z = ax + by + ab$ is
A) $px + qy + z = 0$ B) $px + qy + z^2 = 0$
C) $px - qy = z$ D) $px + qy = z$

- iv) The partial differential equation obtained from $z = e^y f(x+y)$ is
A) $p + z = q$ B) $p - z = q$ C) $p - q = z$ D) None of these

b. Form the partial differential equation by eliminating the arbitrary functions from $z = f(y-2x) + g(2y-x)$. (04 Marks)

c. Solve : $(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$. (06 Marks)

d. Solve : $4\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u$ by the method of separation of variables, given $u(0, y) = 2e^{5y}$.

(06 Marks)

PART - B

5 a. Choose your answers for the following :

(04 Marks)

i) $\int_0^2 \int_0^x (x+y) dx dy = \dots\dots$

A) 0

B) 1

C) 3

D) 4

ii) $\int_0^{\infty} e^{-x^2} dx = \dots\dots$

A) $\sqrt{\pi}$

B) $\frac{\sqrt{\pi}}{2}$

C) $\sqrt{\frac{\pi}{2}}$

D) $\frac{\pi}{2}$

iii) The value of $\beta(2, 1) + \beta(1, 2)$ is

A) 0

B) $\frac{1}{2}$

C) 2

D) 1

iv) $\int_0^2 \int_1^3 \int_1^2 xy^2z \, dz \, dy \, dx = \dots\dots$

A) 26

B) 25

C) 1

D) 0

b. Change the order of integration in $\int_0^1 \int_{x^2}^{1-x} xy \, dx \, dy$ and hence evaluate the same. (04 Marks)

c. Evaluate $\int_0^{\infty} \int_0^{\infty} e^{-(x^2+y^2)} \, dx \, dy$ by changing to polar coordinates. (06 Marks)

d. Show that $\beta(m, n) = \int_0^1 \frac{x^{m-1} + x^{n-1}}{(1+x)^{m+n}} \, dx$. (06 Marks)

6 a. Choose your answers for the following : (04 Marks)

i) If $\vec{F} = x^2\mathbf{i} + xy\mathbf{j}$ then $\int_C \vec{F} \cdot d\vec{r}$ from (0, 0) to (1, 1) along the line $y = x$ is

A) 0

B) $\frac{2}{3}$

C) $\frac{3}{2}$

D) None of these

ii) The value of $\iiint_S (yz \, dy \, dz + zx \, dz \, dx + xy \, dx \, dy)$ where s is the surface of unit sphere $x^2 + y^2 + z^2 = 1$ is

A) 0

B) 4π

C) $\frac{4\pi}{3}$

D) 10π

iii) A necessary and sufficient condition that the line integral $\int_L \vec{F} \cdot d\vec{r}$ for every closed curve C is

A) $\text{Curl } \vec{F} = 0$

B) $\text{div } \vec{F} = 0$

C) $\text{Curl } \vec{F} \neq 0$

D) $\text{div } \vec{F} \neq 0$

iv) If V is the volume bounded by a surface S and \vec{F} is a continuously differentiable vector function then $\iiint_V \text{div } \vec{F} \, dv = \dots\dots$

A) 0

B) $\iint_S \vec{F} \cdot \hat{n} \, ds$

C) $\iint_S \vec{F} \cdot \hat{n} \, ds$

D) None of these

b. Using Green's theorem evaluate $\int_C [(xy + y^2)dx + x^2dy]$ where C is bounded by $y = x$ and $y = x^2$. (04 Marks)

c. Verify Stoke's theorem for the vector $\vec{F} = (x^2 + y^2)\mathbf{i} - 2xy\mathbf{j}$ taken round the rectangle bounded by $x = 0, x = a, y = 0, y = b$. (06 Marks)

d. Using divergence theorem evaluate $\int_S \vec{F} \cdot ds$ where $\vec{F} = 4x\mathbf{i} - 2y^2\mathbf{j} + z^2\mathbf{k}$ and S is the surface bounded by the region $x^2 + y^2 = 4, z = 0, z = 3$. (06 Marks)

- 7 a. Choose your answers for the following : (04 Marks)
- i) If $L\{f(t)\} = f(s)$ then $L\{e^{-at}f(t)\}$ is
 A) $f(s-a)$ B) $f(s+a)$ C) $f(s)$ D) None of these
- ii) $L\left\{\frac{\sin at}{t}\right\} = \dots\dots$
 A) $\cos^{-1}\left(\frac{s}{a}\right)$ B) $\tan^{-1}\frac{s}{a}$ C) $\frac{\pi}{2} + \tan^{-1}\frac{s}{a}$ D) None of these
- iii) $L\{u(t+2)\} = \dots\dots$
 A) $\frac{e^{-2s}}{s^2}$ B) e^{2s} C) $\frac{e^{2s}}{s}$ D) $\frac{e^{-2s}}{s}$
- iv) $L\{s(t)\} = \dots\dots$
 A) 0 B) e^{-as} C) ∞ D) 1
- b. Find the value of $\int_0^{\infty} t^3 e^{-t} \sin t \, dt$ using Laplace transforms. (04 Marks)
- c. If $f(t) = \begin{cases} t, & 0 \leq t \leq a \\ 2a-t, & a \leq t \leq 2a \end{cases}$, where $f(t+2a) = f(t)$, show that $L\{f(t)\} = \frac{1}{s^2} \tan h\left(\frac{as}{2}\right)$. (06 Marks)
- d. Express $f(t) = \begin{cases} 1, & 0 < t \leq 1 \\ t, & 1 < t \leq 2 \\ t^2, & t > 2 \end{cases}$ in terms of unit step function and hence find its Laplace transform. (06 Marks)
- 8 a. Choose your answers for the following : (04 Marks)
- i) $L^{-1}\left\{\frac{1}{s^n}\right\}$ is possible only when n is
 A) zero B) -ve integer C) +ve integer D) -ve rational
- ii) $L^{-1}\left\{\frac{s}{(s-1)^3}\right\} = \dots\dots$
 A) $e^{-t}(t+t^2)$ B) $e^t\left(t+\frac{t^2}{2!}\right)$ C) $t e^t + t^2 e^t$ D) None of these
- iii) $L^{-1}\left\{\log\left(\frac{s+1}{s-1}\right)\right\} = \dots\dots$
 A) $2 \sin t$ B) $2 \cos h t$ C) $\sin h t$ D) $2 \sin h t$
- iv) $L^{-1}\left\{\frac{s}{(2s+3)^2}\right\} = \dots\dots$
 A) $-\frac{1}{8}(2-3t)e^{-\frac{3t}{2}}$ B) $\frac{1}{8}(2-3t)e^{-\frac{3t}{2}}$ C) $2e^{-\frac{3t}{2}} - 3te^{-\frac{3t}{2}}$ D) None of these
- b. Find $L^{-1}\left\{\frac{5s+3}{(s-1)(s^2+2s+5)}\right\}$. (04 Marks)
- c. Using convolution theorem evaluate $L^{-1}\left\{\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right\}$. (06 Marks)
- d. Solve $y''' + 2y'' - y' - 2y = 0$ given $y(0) = y'(0) = 0$ and $y''(0) = 6$ by using Laplace transform method. (06 Marks)
