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10MAT21

Second Semester B.E. Degree Examination, June/July 2011
Engineering Mathematics – II

Time: 3 hrs. Max. Marks:100

Note: 1. *Answer any FIVE full questions, choosing at least two from each part.*
2. *Answer all objective type questions only in OMR sheet page 5 of the answer booklet.*
3. *Answer to objective type questions on sheets other than OMR will not be valued.*

PART – A

1 a. Choose your answers for the following :

i) A differential equation of the first order but of second degree (solvable for P) has the general solution as,

A) $F_1(x,y,c)+F_2(x,y,c)=0$ B) $F_1(x,y,c)\times F_2(x,y,c)=0$

C) $F_1(x,y,c)-F_2(x,y,c)=0$ D) $F_1(x,y,c)/F_2(x,y,c)=0$

ii) If the given differential equation is solving for x then it is of the form,

A) $x=f(P/y)$ B) $y=f(x,P)$ C) $x=f(\frac{y}{P})$ D) $x=f(y,P)$

iii) Clairaut's equation of $P=\sin(y-xP)$ is,

A) $y=\frac{P}{x}+\sin^{-1}P$ B) $y=Px+\sin P$ C) $y=Px+\sin^{-1}P$ D) $y=x+\sin^{-1}P$

iv) The differential equation for R, L series circuit is,

A) $\frac{di}{dt}+Ri=E$ B) $L\frac{di}{dt}+i=E$ C) $\frac{di}{dt}+Ri=\frac{E}{L}$ D) $L\frac{di}{dt}+Ri=E$ (04 Marks)

b. Solve $P(P+y)=x(x+y)$ by solving for P. (05 Marks)

c. Solve $P^3-4xyP+8y^2=0$ by solving for x. (05 Marks)

d. Solve $(Px-y)(Py+x)=a^2P$, use the substitution $X=x^2$, $Y=y^2$. (06 Marks)

2 a. Choose your answers for the following :

i) Roots of $y''-6y'+13y=0$ are,

A) $2\pm3i$ B) $2\pm i$ C) $3\pm i$ D) $3\pm2i$

ii) The value of $\frac{1}{D}(f(x))$ is,

A) $f'(x)$ B) $\frac{1}{f'(x)}$ C) $\int f(x)dx$ D) $\int\frac{1}{f(x)}dx$

iii) The particular integral of $(D^2-6D+9)y=\log 2$ is,

A) $6\log 2$ B) $\frac{1}{9}\log 2$ C) $9\log 2$ D) $\frac{1}{6}\log 2$

iv) The displacement in the simple harmonic motion $\frac{d^2x}{dt^2}=-\mu^2x$ is,

A) $C_1\cos\mu t+C_2\sin\mu t$ B) $C_1\cos\mu t-C_2\sin\mu t$

C) $C_1\cos\mu t\pm C_2\sin\mu t$ D) $\cos\mu t\pm\sin\mu t$ (04 Marks)

b. Solve $(D^3-D)y=2e^x+4\cos x$. (05 Marks)

c. Solve $(D^2+2)y=x^2e^{3x}+\cos 2x$ (05 Marks)

d. Solve the simultaneous differential equations, $\frac{dx}{dt}+5x-2y=t$, $\frac{dy}{dt}+2x+y=0$. (06 Marks)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

3 a. Choose your answers for the following :

i) If y_1 and y_2 are the solutions of second order differential equation and u and v are variation of parameters of $y_p = uy_1 + vy_2$ then v = ___________

A) $\int \frac{(y_1X)dx}{y_1y_2' - y_1'y_2}$ B) $\int \frac{(y_2X)dx}{y_1y_2' + y_1'y_2}$ C) $\int \frac{Xdx}{y_1y_2' - y_1'y_2}$ D) $\int \frac{dx}{y_1y_2' - y_1'y_2}$

ii) In $x^2y'' + 4xy' + 2y = e^x$ if $x = e^t$ then we get for x^2y'' as,

A) (D-1)y B) D(D-1)y C) D(D+1)y D) D(D+2)y

iii) To transform $(ax+b)^2y'' + K_1(ax+b)y' + K_2y = X$ into Legendre's linear equation we put ax+b = ________

A) e^{-t} B) $\frac{1}{e^{-t}}$ C) $1+e^t$ D) $1-e^t$

iv) Series solution is a regular singularity of the equation $P_0y'' + P_1y' + P_2y = 0$ when

A) $x < 0$ B) $x > 0$ C) $x = 0$ D) $x \neq 0$ **(04 Marks)**

b. Solve $y'' - 6y' + 9y = \frac{e^{3x}}{x^2}$ using variation of parameters. **(05 Marks)**

c. Solve $x^2y'' + xy' + y = 2\cos^2(\log x)$. **(05 Marks)**

d. Solve $2xy'' + 3y' - y = 0$ by Frobenius method. **(06 Marks)**

4 a. Choose your answers for the following :

i) Partial differential equation by eliminating a and b from the relation $Z = (x^2+a)(y^2+b)$ is,

A) $Z_xZ_y = xyz$ B) $Z_{xy} = xyz$ C) $Z_{xy} = 4xyz$ D) $Z_xZ_y = 4xyz$

ii) The solution of $Z_{yy} = \sin xy$ is Z = ___________

A) $\sin xy + f(x) + g(y)$ B) $-\frac{1}{x^2}\cos xy + f(x) + g(y)$

C) $-\frac{1}{x^2}\sin xy + yf(x) + g(y)$ D) $-\sin xy + f(x) + xg(y)$

iii) For the Lagrange's linear partial differential equation, Pp+Qq = R, the subsidiary equations are ________

A) $\frac{dx}{P} = \frac{-dy}{Q} = \frac{dz}{R}$ B) $\frac{-dx}{P} = \frac{-dy}{Q} = \frac{dz}{R}$

C) $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$ D) $\frac{dx}{P^2} = \frac{dy}{Q^2} = \frac{dz}{R^2}$

iv) In the method of separation of variables to solve $u_{xx} - 2u_x + u_t = 0$, the trial solution is u = ________

A) X(x)T(t) B) $\frac{X(x)}{T(t)}$ C) $\sqrt{\frac{X(x)}{T(t)}}$ D) $X(x)\sqrt{T(t)}$

(04 Marks)

b. Solve $Z_{xy} = \sin x \sin y$ for which $Z_y = -2\sin y$ when x = 0 and z = 0 when y is an odd multiple of $\frac{\pi}{2}$. **(05 Marks)**

c. Solve $(x^2 - y^2 - z^2)P + 2xyq = 2xz$. **(05 Marks)**

d. Solve $3u_x + 2u_y = 0$, $u(x, 0) = 4e^{-x}$ by the separation of variables. **(06 Marks)**

PART – B

5 a. Choose your answers for the following :

i) The value of $\int_0^1\int_0^6 xydxdy$ is ________.

A) 6 B) 7 C) 8 D) 9

ii) The integral $\int_0^1\int_0^{\sqrt{1-x^2}}(x+y)dydx$ after changing the order of integration is ________

A) $\int_0^2\int_0^{\sqrt{1-y^2}}(x+y)dxdy$ B) $\int_0^1\int_0^{\sqrt{1-y^2}}(x+y)dxdy$ C) $\int_0^1\int_0^{\sqrt{1+y^2}}(x+y)dxdy$ D) $\int_0^1\int_0^{\sqrt{1-y^2}}(x+y)dxdy$

iii) The value of $\int_0^\infty e^{-x^2}dx$ is ________

A) $\pi\sqrt{2}$ B) $2\sqrt{\pi}$ C) $\sqrt{2\pi}$ D) $\frac{\sqrt{\pi}}{2}$

iv) The value of $\Gamma(\frac{1}{4})\Gamma(\frac{3}{4})$ = ______

A) $2\sqrt{\pi}$ B) $\frac{2}{\sqrt{\pi}}$ C) $\pi\sqrt{2}$ D) $\frac{\sqrt{\pi}}{2}$ **(04 Marks)**

b. Evaluate $\int_0^b\int_0^{\frac{a}{b}\sqrt{b^2-y^2}} xydxdy$ by changing the order of integration. **(05 Marks)**

c. Evaluate $\int_{-1}^1\int_0^z\int_{x-z}^{x+z}(x+y+z)dydxdz$ **(05 Marks)**

d. Show that $\int_{-1}^1(1+x)^{m-1}(1-x)^{n-1}dx = 2^{m+n-1}\beta(m,n)$. **(06 Marks)**

6 a. Choose your answers for the following :

i) If $\int_C F.\vec{dr} = 0$ then F is called

A) Rotational B) Solenoidal C) Irrotational D) Dependent

ii) If f is the vector field over a region of volume V in three dimensional space then $\int_V f.dV$ is called

A) Scalar volume integral B) Vector volume integral
C) Scalar surface integral D) Vector surface integral

iii) In Green's theorem in the plane $\iint_A\left(\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}\right)dxdy$ is ________

A) $\int_C(Mdx - Ndy)$ B) $\int_C(Mdx)\times(Ndy)$ C) $\int_C(Ndx - Mdy)$ D) $\int_C(Mdx + Ndy)$

iv) If C be a simple closed curve in space and S be the open surface, f be the vector field then $\int_C f.dr$ = ________

A) $\int_S(\text{curl} f).nds$ B) $\int_S(\nabla\times f).ds$ C) $\int_S(\nabla^2 f).nds$ D) $\int_S(\nabla.f).nds$ **(04 Marks)**

b. Evaluate $\iint_S f.nds$ where $f = yzi + 2y^2j + xz^2k$ and S is the surface of the cylinder $x^2 + y^2 = 9$ contained in the first octant between $z = 0$ and $z = 2$. **(05 Marks)**

c. Verify Green's theorem for $\int_C(xy + y^2)dx + x^2dy$ where C is the closed curve made up of the line $y = x$ and the parabola $y = x^2$. **(05 Marks)**

6 d. Verify Stoke's theorem for $f = (2x - y)i - yz^2 j - y^2 zk$ for the upper half of the sphere $x^2 + y^2 + z^2 = 1$. **(06 Marks)**

7 a. Choose your answers for the following :

i) $L\{\cosh at\} =$ ______

A) $\frac{a}{s^2 + a^2}$ B) $\frac{s}{s^2 - a^2}$ C) $\frac{a}{s^2 - a^2}$ D) $\frac{s}{s^2 + a^2}$

ii) $L\{t^2 e^{-3t}\} =$ ______

A) $\frac{1}{(s+3)^3}$ B) $\frac{2}{(s+3)^2}$ C) $\frac{3}{(s+3)^3}$ D) $\frac{2}{(s+3)^3}$

iii) Transform of unit function $L\{(u(t-a))\} =$ ______

A) $\frac{e^{aS}}{s}$ B) $\frac{e^{-aS}}{s^2}$ C) $\frac{e^{-aS}}{s}$ D) $\frac{e^{aS}}{s^2}$

iv) Unit impulse function $\delta(t-a)$ is $\delta(t-a) = \infty$ for $t = a$; 0 for $t \neq a$ such that $\int_0^\infty \delta(t-a)dt =$ ______

A) 1 B) 0 C) -1 D) $\frac{1}{2}$ **(04 Marks)**

b. Find $L\{t(\sin^3 t - \cos^3 t)\}$. **(05 Marks)**

c. Find $L\{f(t)\}$ when $f(t) = \begin{cases} E, & 0 \le t \le a \\ -E, & a \le t \le 2a \end{cases}$ where the period is 2a. Sketch the graph also. **(05 Marks)**

d. Express f(t) in terms of unit step function and hence find the Laplace transform given that $f(t) = \begin{cases} t^2, & 0 < t < 2 \\ 4t, & 2 < t < 4 \\ 8, & t > 4 \end{cases}$ **(06 Marks)**

8 a. Choose your answers for the following :

i) $L^{-1}\left\{\frac{1}{(s-a)^2 + b^2}\right\} =$ ______

A) $\frac{e^{at}}{b}\cos bt$ B) $\frac{1}{a}e^{at}\sin bt$ C) $\frac{1}{b}\cos bt$ D) $\frac{1}{b}e^{at}\sin bt$

ii) $L^{-1}\left\{\frac{s^2 - 3s + 4}{s^4}\right\} =$ ______

A) $1 - 3t + 2t^3$ B) $1 + \frac{t^2}{3}$ C) $t - \frac{3}{2}t^2 + \frac{2}{3}t^3$ D) $t + \frac{3}{2}t^2 + 1$

iii) In convolution theorem, $L\left\{\int_0^t f(u)g(t-u)du\right\} =$ ______

A) $F(t)G(t)$ B) $F(S) \times G(S)$ C) $\frac{F(S)}{G(S)}$ D) $F(t) - G(t)$

iv) The expression $S^4 L\{x(t)\} - S^3 x(0) - S^2 x'(0) - Sx''(0) - x'''(0)$ is due to,

A) $L\{y'''(t)\}$ B) $L\{x'''(t)\}$ C) $L\{y''(t)\}$ D) $L\{x^{IV}(t)\}$. **(04 Marks)**

b. Find the inverse Laplace transform of $\tan^{-1}\left(\frac{2}{S^2}\right)$. **(05 Marks)**

c. Find $L^{-1}\left\{\frac{s}{(S-1)(S^2+4)}\right\}$ using convolution theorem. **(05 Marks)**

d. Solve $y''(t) + 4y'(t) + 4y(t) = e^t$ with y(0)=0 and $y'(0) = 0$ using Laplace transform method. **(06 Marks)**