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10MAT11

First Semester B.E. Degree Examination, June 2012
Engineering Mathematics – I

Time: 3 hrs. Max. Marks:100

Note: 1. ***Answer any FIVE full questions, choosing at least two from each part.***
2. Answer all objective type questions only on OMR sheet page 5 of the answer booklet.
3. Answer to objective type questions on sheets other than OMR will not be valued.

PART – A

1 a. Choose your answers for the following :

i) The n^{th} derivative of $\cos^2 x$ is

A) $2^{n-1}\cos\left(2x+\frac{n\pi}{2}\right)$ B) $2^n\cos\left(2x+\frac{n\pi}{2}\right)$ C) $2^{n-1}\cos(2x+n\pi)$ D) $2^{n-1}\cos\left(\frac{n\pi}{2}\right)$

ii) The value of C of the Cauchy mean value theorem for $f(x) = e^x$ and $g(x) = e^{-x}$ in [4, 5] is

A) $\frac{5}{2}$ B) $\frac{3}{2}$ C) $\frac{9}{2}$ D) $\frac{1}{2}$

iii) Find the n^{th} derivative of $y = x^{n-1}\log x$ is

A) $y_n = \frac{(n+1)!}{x}$ B) $y_n = \frac{n!}{x}$ C) $y_n = \frac{(n-1)!}{x}$ D) $y_n = \frac{n!}{x^2}$

iv) Maclaurin's series expansion of log (1 + x) is

A) $x+\frac{x^3}{2}+\frac{x^5}{5}+\ldots\ldots$ B) $x-\frac{x^3}{3}+\frac{x^4}{5}-\ldots\ldots$

C) $x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{5}+\ldots\ldots$ D) $x+\frac{x^2}{3}+\frac{x^3}{16}+\ldots\ldots$ **(04 Marks)**

b. By informing in two different ways the nth derivative of x^{2n}, prove that
$$1+\frac{n^2}{1^2}+\frac{n^2(n-1)^2}{1^2\times 2^2}+\frac{n^2(n-1)^2(n-2)^2}{1^2\times 2^2\times 3^2}+\ldots\ldots\ldots=\frac{(2n)!}{(n!)^2}$$ **(06 Marks)**

c. Verify Rolle's theorem for the function $f(x)=\frac{\sin 2x}{e^{2x}}$ in $\left[0,\frac{\pi}{2}\right]$. **(04 Marks)**

d. Using Maclaurin's series, expand log sec x upto the term containing x^6. **(06 Marks)**

2 a. Choose your answers for the following :

i) The value of $\lim_{x\to\infty}\frac{\log x}{x}$ is

A) 0 B) 1 C) 2 D) –2

ii) If s is the arc length of the curve x = f(y) then $\frac{ds}{dy}$ is

A) $\sqrt{1+y_1^2}$ B) $\sqrt{1+y_1}$ C) $\sqrt{1+\left(\frac{dx}{dy}\right)^2}$ D) $\sqrt{\left(\frac{dy}{dx}\right)^2+\left(\frac{dx}{dy}\right)^2}$

iii) Pedal equation to the curve $\frac{2a}{r}=1-\cos\theta$ is

A) $P = ar^2$ B) $P^2 = a^2r$ C) $P^2 = a^2r^2$ D) $P^2 = ar$

iv) The angle between two curves $r = ae^\theta$ and $re^\theta = b$ is

A) $\frac{\pi}{2}$ B) $\frac{\pi}{4}$ C) 0 D) π **(04 Marks)**

2 b. For the curve $y = \frac{ax}{a+x}$, if ρ is the radius of curvature at any point (x, y), show that:

$$\left(\frac{2\rho}{a}\right)^{2/3} = \left(\frac{y}{x}\right)^2 + \left(\frac{x}{y}\right)^2$$ **(06 Marks)**

c. Evaluate $\lim_{x \to 0} \left(\frac{\sin x}{x}\right)^{1/x^2}$. **(04 Marks)**

d. Find the angle between the curves $r = \frac{a}{1+\cos\theta}$; $r = \frac{b}{1-\cos\theta}$. **(06 Marks)**

3 a. Choose your answers for the following :

i) When $u = y^2 \log\left(\frac{x}{y}\right)$ then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ is

A) u B) u^2 C) 2u D) 3u

ii) The Taylor's series of f(x, y) = xy at (1, 1) is

A) $1 + [(x-1)+(y-1)]$ B) $1 + [(x-1)+(y-1)] + [(x-1)(y-1)]$

C) $[(x-1)(y-1)]$ D) None of these

iii) The Jacobian of transformation from the Cartesian to polar coordinates system is

A) r^3 B) r C) r^2 D) r sin θ

iv) The rectangular solid of maximum volume which can be inscribed in a sphere is

A) parallelogram B) rectangle C) cube D) triangle. **(04 Marks)**

b. Examine the function $\sin x + \sin y + \sin(x+y)$ for extreme values. **(06 Marks)**

c. Find the possible error in percent in computing the parallel resistance 'r' of two resistances r_1 and r_2 from the formula $\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2}$ are both in error by 2%. **(04 Marks)**

d. If $z(x+y) = x^2 + y^2$ show that $\left[\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right]^2 = 4\left[1 - \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right]$. **(06 Marks)**

4 a. Choose your answers for the following :

i) A gradient of the scalar point function φ that is ∇φ is

A) vector function B) scalar function C) zero D) φ

ii) The directional derivative of $f(x, y, z) = x^2yz + 4xz^2$ at the (1, −2, −1) in the direction PQ where P = (1, 2, −1), Q = (−1, 2, 3) is

A) $\frac{28}{\sqrt{5}}$ B) $\frac{30}{\sqrt{4}}$ C) $\frac{-28}{\sqrt{5}}$ D) $\frac{20}{\sqrt{6}}$

iii) If $\overline{R}$ is the position vector of any point P(x, y, z) then $\nabla \cdot \overline{R}$ is

A) 3 B) −3 C) 2 D) 0

iv) If $\overline{r} = x\hat{i} + y\hat{j} + z\hat{k}$ then Curl $\overline{r}$ = ...

A) 0 B) 1 C) −1 D) ∞ **(04 Marks)**

b. Find the constants a and b such that $\overline{F} = (axy + z^3)i + (3x^2 - z)j + (bxz^2 - y)k$ is irrotational and find scalar potential function φ such that F = ∇φ. **(06 Marks)**

c. Prove that $\nabla x\left[\frac{a x \overline{r}}{r^n}\right] = \frac{-\overline{a}}{r^3} + \frac{3(a.\overline{r})\overline{r}}{r^5}$. **(04 Marks)**

d. Prove that the cylindrical coordinates system is orthogonal. **(06 Marks)**

PART – B

5 a. Choose your answers for the following :

i) The value of $\int_0^1 x^2(1-x^2)^{3/2}dx$ is

A) $\frac{\pi}{23}$ B) $\frac{1}{32}$ C) $\frac{\pi}{32}$ D) $\frac{\pi}{16}$

ii) The tangent to the curve $y^2 = 4ax$ at origin is

A) y-axis B) x-axis
C) both x-axis and y-axis D) does not exist

iii) The value of $\int_0^{\pi} \sin^4\left(\frac{x}{2}\right)dx$ is

A) $\frac{3\pi}{18}$ B) $\frac{3\pi}{8}$ C) $\frac{3\pi}{16}$ D) $\frac{3\pi^2}{8}$

iv) The surface area of the sphere of radius 'a' is

A) $4\pi a^2$ B) $4\pi^2 a$ C) $4\pi a$ D) $2\pi a^2$ **(04 Marks)**

b. Obtain the reduction formula for $\int \sin^m x \cos^n x\, dx$. **(06 Marks)**

c. Evaluate $\int_0^{\infty} \frac{\tan^{-1}(ax)}{x(1+x^2)}dx$ using the method of differentiation under integral sign. **(04 Marks)**

d. Find the area of the loop of the curve $ay^2 = x^2(a-x)$. **(06 Marks)**

6 a. Choose your answers for the following :

i) The solution of the differential equation $\frac{dy}{dx} = e^{x-y}$ is

A) $e^x + e^{-y} = c$ B) $e^{-x} + e^{-y} = c$ C) $e^x + e^y = c$ D) $e^{x+y} = c$

ii) If the homogeneous differential equation $\frac{dy}{dx} = \frac{f_1(x,y)}{f_2(x,y)}$ the degree of the homogeneous functions $f_1(x,y)$ and $f_2(x,y)$ are

A) different B) same
C) relatively prime D) degree of $f_1(x,y) >$ degree of $f_2(x,y)$

iii) The integrating factor of the differential equation $(1+x^2)\frac{dy}{dx} + xy = \sinh^{-1}x$ is

A) $\frac{1}{\sqrt{1+x^2}}$ B) $\sqrt{1-x^2}$ C) $\sqrt{1+x^2}$ D) $\frac{x}{\sqrt{1+x^2}}$

iv) If replacing $\frac{dy}{dx}$ by $-\frac{dx}{dy}$ in the differential equation $f\left(x, y, \frac{dy}{dx}\right) = 0$ we get the differential equation of

A) polar trajectory B) orthogonal trajectory
C) parametric trajectory D) parallel trajectory **(04 Marks)**

b. Solve $(1+xy^2)\frac{dy}{dx} = 1$. **(06 Marks)**

c. Solve $\frac{dy}{dx} = \frac{x(2\log x + 1)}{\sin y + y\cos y}$. **(04 Marks)**

d. Find the orthogonal trajectory of $r^n = a^n \sin n\theta$ **(06 Marks)**

7 a. Choose your answers for the following :

i) In a system of linear equations if the rank of the co-efficient matrix = rank of the augmented matrix = n number of unknowns then the system has

A) no solutions B) unique solutions

C) infinite number of solutions D) trivial solutions

ii) The rank of matrix $\begin{bmatrix} 2 & -1 & 3 & 1 \\ 1 & 4 & -2 & 1 \\ 5 & 2 & 4 & 3 \end{bmatrix}$ is

A) 3 B) 4 C) 2 D) 5

iii) A square matrix in which aij = aji for all i and j then it is called a

A) unique matrix B) symmetric matrix C) skew symmetric D) triangular matrix

iv) The inverse of the square matrix A is

A) $|A|$ B) $\frac{\text{adj}\,A}{|A|}$ C) adj A D) $\frac{|A|}{\text{adj}\,A}$ **(04 Marks)**

b. Investigate for what value of λ and μ the simultaneous equation $x+y+z=6$, $x+2y+3z=10$, $x+2y+\lambda z=\mu$ have

i) no solutions ii) unique solutions iii) infinite number of solutions. **(06 Marks)**

c. Apply Gauss-elimination method to solve the following equations:

$2x-y+3z=1$, $-3x+4y-5z=0$, $x+3y-6z=0$ **(04 Marks)**

d. Find the rank of $\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$. **(06 Marks)**

8 a. Choose your answers for the following :

i) The eigen values of the matrix $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ are

A) 2, 3, 8 B) 2, 2, 8 C) 8, 4, 3 D) 2, −2, 8

ii) A homogeneous expression of the second degree in any number of variables is called a

A) quadratic form B) diagonal form C) symmetric form D) spectral form

iii) A square matrix A of order 3 has 3 linearly independent eigen vectors then a matrix P can be found such that $P^{-1}AP$ is a

A) diagonal matrix B) symmetric matrix C) unit matrix D) singular matrix

iv) If the eigen vector is (1, 1, 1) then its normalized form is

A) $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$ B) $\left(\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}\right)$

C) $\left(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$ D) $\left(-\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$ **(04 Marks)**

b. Reduce $6x^2+3y^2-4xy-2yz+4zx$ into canonical form. **(06 Marks)**

c. Find all the eigen values for the matrix, $A = \begin{bmatrix} 7 & -2 & 0 \\ -2 & 6 & -2 \\ 0 & -2 & 5 \end{bmatrix}$. **(04 Marks)**

d. Reduce the matrix, $A = \begin{bmatrix} 11 & -4 & 7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{bmatrix}$ into a diagonal matrix. **(06 Marks)**

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