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06CS34

Third Semester B.E. Degree Examination, June/July 2011
Discrete Mathematical Structures

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. For any three sets A, B and C, prove the following :

$A-(B\cup C)=(A-C)-(B-C)=(A-C)-B.$ **(08 Marks)**

b. A professor has tow dozen introductory test books on computer science. He is concerned about their coverage of the topics (A) compliers, (B) data structures and (C) operating systems. Following data are the number of books which contain material on these topics.

$|A|=8$, $|B|=13$, $|C|=13$, $|A\cap B|=5$, $|A\cap C|=3$, $|B\cap C|=6$ and $|A\cap B\cap C|=2$. Determine :

i) The number of test books which include material on exactly one of these topics.

ii) The number of text books which do not deal with any of the topics. **(06 Marks)**

c. An integer is selected at random from 3 through 17 inclusive. If A is the event that a number divisible by 3 is chosen and B is the event that the number exceeds 10, determine

Pr(A), Pr(B), $Pr(A\cap B)$ and $Pr(A\cup B)$.How is $Pr(A\cup B)$ related to Pr(A), Pr(B) and $Pr(A\cap B)$? **(06 Marks)**

2 a. Establish the following :

$$\begin{array}{l} P \\ p \to q \\ s \vee r \\ r \to \neg q \\ \hline \therefore s \vee t. \end{array}$$

(06 Marks)

b. Show that each of the following arguments is invalid by providing a counter example – that is an assignment of truth values for the given primitive statements such that all premises are time while the conclusion is false.

i)
$$\begin{array}{l} p \wedge \neg q \\ p \to (q \to r) \\ \hline \therefore \neg r \end{array}$$

ii)
$$\begin{array}{l} P \leftrightarrow q \\ q \to r \\ r \vee \neg s \\ \neg s \to q \\ \hline \therefore s. \end{array}$$

(08 Marks)

c. Consider each of the following arguments. If the argument is valid, identify the rule inference which establishes its validity. If not, indicate whether the error is due to an attempt to argue by converse or by the inverse.

i) If Gopal's computer program is correct, then he will be able to complete his computer science assignment in at most three hours

ii) It takes Gopal over three hours to complete his computer science assignment. Therefore Gopal's computer program is not correct.

iii) If interest rates fall, then the stock market will rise. Interest rates are not falling. Therefore the stock market will not rise. **(06 Marks)**

3 a. For the following statement. State the converse, inverse and contrapositive. Determine the truth value of the given statement and the truth values of its converse, inverse and contrapositive. The universe consists of all integers.
If m divides n and n divides p, then m divides p. **(06 Marks)**

b. Establish the validity of the following argument

$(x)\,[p(x) \vee q(x)]$
$(x)[(\neg p(x) \wedge q(x) \rightarrow r(x)]$

$\therefore (x)[\neg r(x) \rightarrow p(x)]$ **(08 Marks)**

c. Let n be an integer. Prove that n is odd if and only if $\neg n + 8$ is odd. **(06 Marks)**

4 a. Prove the following using the principle of mathematical induction

i) $1^2 + 3^2 + 5^2 + \cdots + (2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}$

ii) $1 \times 3 + 2 \times 4 + 3 \times 5 + \cdots + n(n+2) = \frac{n(n+1)(2n+7)}{6}$. **(08 Marks)**

b. Prove that any positive integer greater than or equal to 14 can be expressed as sum of 3s and / or 8s. **(06 Marks)**

c. Fibonacci members are defined recursively as follows
$F_0 = 0$, $F_1 = 1$ and $F_n = F_{n-1} + F_{n-2}$ for $n \in Z^+$ with $n \geq 2$. Show that $\sum_{i=0}^{n} F_i^2 = F_n \times F_{n+1}$ for all $n \in Z^+$, where Z^+ denotes the set of all positive integers. **(06 Marks)**

PART - B

5 a. Consider two functions $f : A \rightarrow B$ and $g : B \rightarrow C$. Prove that if $g \circ f : A \rightarrow C$ is one –one, then f is one – one and if $gof : A \rightarrow C$ is onto, then g is onto. **(07 Marks)**

b. Let $f, g, h : Z \rightarrow z$ be defined as $f(x) = x - 1$, $g(x) = 2x$ and

$$h(x) = \begin{cases} 7 & \text{if } x \text{ is even} \\ 4 & \text{if } x \text{ is odd} \end{cases}$$

Determine :
i) $f \circ g$
ii) $g \circ f$
iii) $g \circ h$
iv) $h \circ g$
v) $f \circ (g \circ h)$
vi) $(f \circ g) \circ h$. **(06 Marks)**

c. Show that if any 14 integers are selected from the set $S = \{1, 2, 3, \cdots, 25\}$, there are at least two integers whose sum is 26. **(07 Marks)**

6 a. Let A be a finite set which consists of n elements. Determine the number of relations on A which are i) reflexive ii) symmetric iii) antisymmetric. **(08 Marks)**

b. Let A = {a, b, c, d, e, f, g} and consider the partition
P = {{a, c, d}, {b}, {e, g}, {f}}. Determine the corresponding equivalence relation R. **(03 Marks)**

c. For the projects whose Hasse diagrams are given in Fig.Q6(c)(i) and (ii), find maximal elements and minimal elements (if they exit). **(04 Marks)**

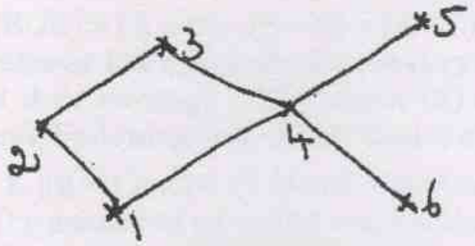

Fig. Q6(c)(i)

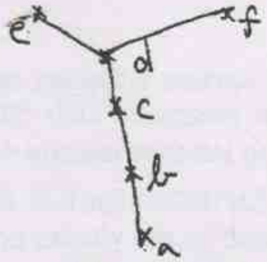

Fig. Q6(c)(ii)

d. Consider the poset (A, $\leq$) where A = {a, b, c, d, e, f, g, h} and $\leq$ is given by the following Hasse diagram, shown in Fig.Q6(d).

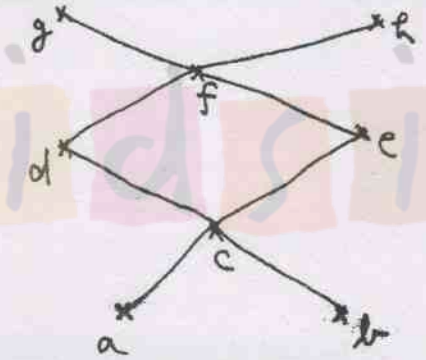

Fig. Q6(d)

Consider the subset B = {c, d, e}. If upper bounds and lower bounds of B exist find them. If they exist, determine the least upper bound (ℓub) and greatest lower bound (gℓb). **(05 Marks)**

7 a. Let $\mathbb{R}^*$ denote the set of all non zero real numbers and let $S = \mathbb{R}^* \times \mathbb{R}$. Define an operation o on S as (u, v) o (x, y) = (ux, vx + y). Prove that (S, o) is a nonableian group. **(07 Marks)**

b. State and prove Lagrange's theorem. **(07 Marks)**

c. For a group G, prove that the function $f : G \rightarrow G$ defined as $f(a) = a^{-1}$ is an isomoraphism if and only if G is abelian. **(06 Marks)**

8 a. In a group code, prove that the minimum distance between distinct code words is the minimum of the weights of the non zero elements of the code. **(06 Marks)**

b. Let (R, +, *) be a ring such that a * a = a for all $a \in R$. Prove the following :
i) a + a = 0 for all $a \in R$, where 0 denotes the identity element of (R, +)
ii) * is commutative. **(08 Marks)**

c. Prove that a field is an integral domain. Give an example of an integral domain which is not a field. **(06 Marks)**

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