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**Third Semester B.E. Degree Examination, June 2012**

**Discrete Mathematical Structures**

Time: 3 hrs.

Max. Marks:100

**Note: Answer FIVE full questions, selecting at least TWO questions from each part.**

**PART - A**

- 1 a. Let  $S = \{21, 22, 23, \dots, 39, 40\}$ . Determine the number of subsets  $A$  of  $S$  such that :
- $|A| = 5$
  - $|A| = 5$  and the largest element in  $A$  is 30
  - $|A| = 5$  and the largest element is at least 30
  - $|A| = 5$  and the largest element is at most 30
  - $|A| = 5$  and  $A$  consists only of odd integers. (10 Marks)
- b. Prove or disprove: For non-empty sets  $A$  and  $B$ ,  $P(A \cup B) = P(A) \cup P(B)$  where  $P$  denotes power set. (05 Marks)
- c. In a group of 30 people, it was found that 15 people like Rasagulla, 17 like Mysorepak, 15 like Champakali, 8 like Rasagulla and Mysorepak, 11 like Mysorepak and Champakali, 8 like Champakali and Rasagulla and 5 like all three. If a person is chosen from this group, what is the probability that the person will like exactly 2 sweets? (05 Marks)
- 2 a. Verify that  $[p \rightarrow (q \rightarrow r)] \rightarrow [(p \rightarrow q) \rightarrow (p \rightarrow r)]$  is a tautology. (05 Marks)
- b. Write dual, negation, converse, inverse and contrapositive of the statement given below :  
If Kabir wears brown pant, then he will wear white shirt. (05 Marks)
- c. Define  $(p \uparrow q) \Leftrightarrow \neg(p \wedge q)$ . Represent  $p \vee q$  and  $p \rightarrow q$  using only  $\uparrow$ . (05 Marks)
- d. Establish the validity or provide a counter example to show the invalidity of the following arguments : (05 Marks)
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| i) $p \vee q$<br>$\neg p \vee r$<br>$\neg r$<br><hr style="width:50%; margin-left:0;"/> $\therefore q$ | ii) $p$<br>$p \rightarrow r$<br>$p \rightarrow (q \vee \neg r)$<br><hr style="width:50%; margin-left:0;"/> $\neg q \vee \neg s$<br>$\therefore s$ |
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- 3 a. For the universe of all polygons with three or four sides, define the following open statements:
- $i(x)$ : all the interior angles of  $x$  are equal  
 $h(x)$ : all sides of  $x$  are equal  
 $s(x)$ :  $x$  is a square  
 $t(x)$ :  $x$  is a triangle
- Translate each of the following statements into an English sentence and determine its truth value:
- $\forall x [s(x) \leftrightarrow (i(x) \wedge h(x))]$
  - $\exists x [t(x) \rightarrow (i(x) \leftrightarrow h(x))]$
- Write the following statements symbolically and determine their truth values.
- Any polygon with three or four sides is either a triangle or a square
  - For any triangle if all the interior angles are not equal, then all its sides are not equal. (08 Marks)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.  
2. Any revealing of identification, appeal to evaluator and/or equations written eg.  $42+8=50$ , will be treated as malpractice.

- 3 b. Let  $p(x, y)$  denote the open statement  $x$  divides  $y$  where the universe consists of all integers. Determine the truth values of the following statements. Justify your answers.
- i)  $\forall x \forall y [p(x, y) \wedge p(y, x) \rightarrow (x = y)]$       ii)  $\forall x \forall y [p(x, y) \vee p(y, x)]$       (06 Marks)
- c. Prove that for every integer  $n$ ,  $n^2$  is even if and only if  $n$  is even.      (06 Marks)
- 4 a. Prove  $\sum_{i=1}^n \frac{1}{i(i+1)} = \frac{n}{n+1} \quad \forall n \in \mathbb{Z}^+$ .      (06 Marks)
- b. Prove  $2^n < n! \quad \forall n > 3$  and  $n \in \mathbb{Z}^+$ .      (06 Marks)
- c. Define an integer sequence recursively by
- $$a_0 = a_1 = a_2 = 1$$
- $$a_n = a_{n-1} + a_{n-3} \quad \forall n \geq 3.$$
- Prove that  $a_{n+2} \geq (\sqrt{2})^n \quad \forall n \geq 0$ .      (08 Marks)

### PART - B

- 5 Let  $A = \{\alpha, \beta, \gamma\}$ ,  $B = \{\theta, \eta\}$ ,  $C = \{\lambda, \mu, \nu\}$ .
- a. Find  $(A \cup B) \times C$ ,  $A \cup (B \times C)$ ,  $(A \times B) \cup C$  and  $A \times (B \cup C)$ .      (08 Marks)
- b. Give an example of a relation from  $A$  to  $B \times B$  which is not a function.      (04 Marks)
- c. How many onto functions are there from (i)  $A$  to  $B$ , (ii)  $B$  to  $A$ ?      (02 Marks)
- d. i) Write a function  $f: A \rightarrow C$  and a function  $g: C \rightarrow A$ . Find  $g \circ f: A \rightarrow A$ .  
ii) Write an invertible function  $f: A \rightarrow C$  and find its inverse.      (06 Marks)
- 6 a. Let  $A = \{1, 2, 3, 4\}$ ,  $B = \{w, x, y, z\}$  and  $C = \{p, q, r, s\}$ . Consider  $R_1 = \{(1, x), (2, w), (3, z)\}$  a relation from  $A$  to  $B$ ,  $R_2 = \{(w, p), (z, q), (y, s), (x, p)\}$  a relation from  $B$  to  $C$ .
- i) What is the composite relation  $R_1 \circ R_2$  from  $A$  to  $C$ ?  
ii) Write relation matrices  $M(R_1)$ ,  $M(R_2)$  and  $M(R_1 \circ R_2)$   
iii) Verify  $M(R_1) \cdot M(R_2) = M(R_1 \circ R_2)$       (06 Marks)
- b. Let  $A = \{1, 2, 3, 6, 9, 12, 18\}$  and define a relation  $R$  on  $A$  as  $xRy$  iff  $x|y$ . Draw the Hasse diagram for the poset  $(A, R)$ .      (06 Marks)
- c. Let  $A = \{1, 2, 3, 4, 5\} \times \{1, 2, 3, 4, 5\}$  and define  $R$  as  $(x_1, y_1)R(x_2, y_2)$  iff  $x_1 + y_1 = x_2 + y_2$ .
- i) Verify that  $R$  is an equivalence relation on  $A$ .  
ii) Determine the equivalence class  $[(1, 3)]$ .  
iii) Determine the partition induced by  $R$ .      (08 Marks)
- 7 a. Define a binary operation  $*$  on  $\mathbb{Z}$  as  $x * y = x + y - 1$ . Verify that  $(\mathbb{Z}, *)$  is an abelian group.      (07 Marks)
- b. Let  $f: G \rightarrow H$  be a group homomorphism onto  $H$ . If  $G$  is an abelian group, prove that  $H$  is also abelian.      (07 Marks)
- c. The encoding function  $E: \mathbb{Z}_2^2 \rightarrow \mathbb{Z}_2^5$  is given by the generator matrix  $G = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix}$ .
- i) Determine all the code words.  
ii) Find the associated parity-check matrix  $H$ .      (06 Marks)
- 8 a. If  $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \in M_2(\mathbb{R})$ , prove that  $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$  is a unit of this ring if and only if  $ad - bc \neq 0$ .      (08 Marks)
- b. Let  $R$  be a ring with unity and  $a, b$  be units in  $R$ . Prove that  $ab$  is a unit of  $R$  and that  $(ab)^{-1} = b^{-1}a^{-1}$ .      (06 Marks)
- c. Find multiplicative inverse of each (non-zero) element of  $\mathbb{Z}_7$ .      (06 Marks)

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