

# First/Second Semester B.E. Degree Examination, June/July 2013

## Engineering Mathematics – I

Time: 3 hrs.

Max. Marks: 100

- Note:** 1. Answer any FIVE full questions, choosing at least two from each part.  
2. Answer all objective type questions only on OMR sheet page 5 of the answer booklet.  
3. Answer to objective type questions on sheets other than OMR will not be valued.

### PART – A

- 1 a. Choose your answers for the following : (04 Marks)
- i) If  $y = 3^{5x}$  then  $y_n$  is      A)  $(3 \log 5)^n e^{5x}$     B)  $(5 \log 3)^n e^{5x}$     C)  $(5 \log 3)^{-n} e^{5x}$     D)  $(5 \log 3)^n e^{-5x}$
- ii) If  $y = \cos^2 x$  then  $y_n$  is  
A)  $2^{n+1} \cos(n\pi/2 + 2x)$     B)  $2^{n-1} \cos(n\pi/2 + 2x)$     C)  $2^{n-1} \cos(n\pi/2 - 2x)$     D)  $2^{n+1} \cos(n\pi/2 - 2x)$
- iii) The Lagrange's mean value theorem for the function  $f(x) = e^x$  in the interval  $[0, 1]$  is  
A)  $C = 0.5413$       B)  $C = 2.3$       C)  $0.3$       D) None of these
- iv) Expansion of  $\log(1 + e^x)$  in powers of  $x$  is \_\_\_\_\_.  
A)  $\log 2 - x/2 + x^2/8 + x^4/192 + \dots$   
B)  $\log 2 + x/2 + x^2/8 - x^4/192 + \dots$     C)  $\log 2 + x/2 + x^2/8 + x^4/192 + \dots$     D)  $\log 2 - \frac{x}{2} - \frac{x^2}{8} - \frac{x^4}{192} + \dots$
- b. If  $y^{1/m} + y^{-1/m} = 2x$  prove that  $(x^2 - 1)y_{n+2} + (2n + 1)xy_{n+1} + (n^2 - m^2)y_n = 0$ . (06 Marks)
- c. Verify the Rolle's theorem for the functions :  $f(x) = e^x(\sin x - \cos x)$  in  $(\pi/4, 5\pi/4)$ . (06 Marks)
- d. By using Maclaurin's theorem expand  $\log \sec x$  up to the term containing  $x^6$ . (04 Marks)
- 2 a. Choose your answers for the following : (04 Marks)
- i) The indeterminate form of  $\lim_{x \rightarrow 0} \frac{a^x - b^x}{x}$  is    A)  $\log(b/a)$     B)  $\log(a/b)$     C) 1    D) -1
- ii) The angle between the radius vector and the tangent for the curves  $r = a(1 - \cos \theta)$  is  
A)  $\theta/2$       B)  $-\theta/2$       C)  $\pi/2 + \theta$       D)  $\pi/2 - \theta/2$
- iii) The polar form of a curve is \_\_\_\_\_.    A)  $r = f(\theta)$     B)  $\theta = f(y)$     C)  $r = f(x)$     D) None of these
- iv) The rate at which the curve is bending called \_\_\_\_\_.    A) Radius of curvature;    B) Curvature;    C) Circle of curvature;    D) Evaluate.
- b. Evaluate  $\lim_{x \rightarrow 0} \left( \frac{\sin x}{x} \right)^{1/x^2}$ . (06 Marks)
- c. Find the angles of intersection of the following pairs of curves,  $r = a\theta/(1 + \theta)$ ;  $r = a/(1 + \theta^2)$ . (06 Marks)
- d. Find the radius of curvature at  $(3a/2, 3a/2)$  on  $x^3 + y^3 = 3axy$ . (04 Marks)
- 3 a. Choose your answers for the following : (04 Marks)
- i) If  $u = x^2 + y^2$  then  $(\partial^2 u)/(\partial x \partial y)$  is equal to      A) 2      B) 0      C) 2x      D) 2y
- ii) If  $z = f(x, y)$  where  $x = u - v$  and  $y = uv$  then  $(u + v)(\partial z / \partial x)$  is  
A)  $u(\partial z / \partial u) - v(\partial z / \partial v)$     B)  $u(\partial z / \partial u) + v(\partial z / \partial v)$     C)  $\partial z / \partial u + \partial z / \partial v$     D)  $\partial z / \partial u - \partial z / \partial v$
- iii) If  $x = r \cos \theta$ ,  $y = r \sin \theta$  then  $[\partial(r, \theta)]/[\partial(x, y)]$  is      A)  $r$       B)  $1/r$       C) 1      D) -1
- iv) In errors and approximations  $\partial x/x$ ,  $\partial y/y$ ,  $\partial f/f$  are called  
A) relative error      B) percentage error      C) error in  $x, y$  and  $f$       D) none of these
- b. If  $x^x y^y z^z = c$ , show that  $\partial^2 z / \partial x \partial y = -[x \log ex]^{-1}$ , when  $x = y = z$ . (06 Marks)
- c. Obtain the Jacobian of  $\partial(x, y, z)/\partial(r, \theta, \phi)$  for change of coordinate from three dimensional Cartesian coordinates to spherical polar coordinates. (06 Marks)
- d. In estimating the cost of a pile of bricks measured as  $2m \times 15m \times 1.2m$ , the tape is stretched +1% beyond the standard length. If the count is 450 bricks to 1 cu.cm and bricks cost of 530 per 1000, find the approximate error in the cost. (04 Marks)
- 4 a. Choose your answers for the following : (04 Marks)
- i) If  $\vec{R} = xi + yj + zk$  then  $\text{div } \vec{R}$       A) 0      B) 3      C) -3      D) 2
- ii) If  $\vec{F} = 3x^2 i - xyj + (a - 3)xz k$  is Solenoidal then  $a$  is equal to \_\_\_\_    A) 0    B) -2    C) 2    D) 3
- iii) If  $\vec{F} = (x + y + 1)i + j - (x + y)k$  then  $\vec{F} \cdot \text{curl } \vec{F}$  is \_\_\_\_\_.    A) 0    B)  $x + y$     C)  $x + y + z$     D)  $x - y$
- iv) The scale factors for cylindrical coordinate system  $(\rho, \phi, z)$  are given by  
A)  $(\rho, 1, 1)$       B)  $(1, \rho, 1)$       C)  $(1, 1, \rho)$       D) none of these
- b. Prove that  $\text{curl } \vec{A} = \text{grad}(\text{div } \vec{A}) - \nabla^2 \vec{A}$ . (06 Marks)
- c. Find the constants  $a, b, c$  such that the vector  $\vec{F} = (x + y + az)i + (bx + 2y - z)j + (x + cy + 2z)k$  is irrotational. (06 Marks)
- d. Derive an expression for  $\nabla \cdot \vec{A}$  in orthogonal curvilinear coordinates. Deduce  $\nabla \cdot \vec{A}$  is rectangular coordinates. (04 Marks)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.  
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

**PART - B**

**10MAT11**  
(04 Marks)

- 5 a. Choose your answers for the following :
- i) The value of  $\int_0^{\infty} e^{-\alpha x} dx$  is \_\_\_\_ A)  $1/e$  B)  $-1/e$  C)  $1/\alpha$  D)  $-1/\alpha$
- ii) The value of the integral  $\int_0^{\pi/2} \sin^7 x dx$  is A)  $35/16$  B)  $16/35$  C)  $-16/35$  D)  $18/35$
- iii) The volume generated by revolving the cardioid  $r = a(1 + \cos \theta)$  about the initial line is  
A)  $(3\pi a^2)/8$  B)  $(3\pi a^3)/8$  C)  $(2\pi a^2)/9$  D) None
- iv) The area of the loop of the curve  $r = a \sin 3\theta$  is \_\_\_\_ A)  $a^2/12$  ; B)  $\pi/12$  ; C)  $\pi a^2/12$  ; D) None
- b. By applying differential under the integral sign evaluate  $\int_0^{\pi/2} \frac{\log(1 + y \sin^2 x)}{\sin^2 x} dx$ . (06 Marks)
- c. Evaluate of  $\int_0^{\pi/2} \sin^n x dx$  where n is any integer. (06 Marks)
- d. Find the length of the arch of the cycloid  $x = a(\theta - \sin \theta)$ ;  $y = a(1 - \cos \theta)$ ;  $0 < \theta \leq 2\pi$ . (04 Marks)
- 6 a. Choose your answers for the following : (04 Marks)
- i) The general solution of the differential equation  $(dy/dx) = (y/x) + \tan(y/x)$  is  
A)  $\sin(y/x) = c$  B)  $\sin(y/x) = cx$  C)  $\cos(y/x) = cx$  D)  $\cos(y/x) = c$
- ii) An integrating factor for  $ydx - xdy = 0$  is A)  $x/y$  B)  $y/x$  C)  $1/(x^2y^2)$  D)  $1/(x^2+y^2)$
- iii) The differential equation satisfying the relation  $x = A \cos(mt - \alpha)$  is  
A)  $(dx/dt) = 1 - x^2$  B)  $(d^2x/dt^2) = -\alpha^2 x$  C)  $(d^2x/dt^2) = -m^2 x$  D)  $(dx/dt) = -m^2 x$
- iv) The orthogonal trajectories of the system given by  $r = a\theta$  is  
A)  $r^2 = ke^\theta$  B)  $r = ke^\theta$  C)  $r^2 e^{-\theta^2} = k$  D)  $r^2 = k e^{-\theta^2}$
- b. Solve  $(x \cos(y/x) + y \sin(y/x))y - (y \sin(y/x) - x \cos(y/x))x (dy/dx) = 0$ . (06 Marks)
- c. Solve  $(1 + y^2) + (x - e^{\tan^{-1} y}) dy/dx = 0$ . (06 Marks)
- d. Prove that the system of parabola  $y^2 = 4a(x + a)$  is self orthogonal. (04 Marks)
- 7 a. Choose your answers for the following : (04 Marks)
- i) Find the rank of  $\begin{bmatrix} 3 & -1 & 2 \\ -6 & 2 & 4 \\ -3 & 1 & 2 \end{bmatrix}$ : A) 3 B) 2 C) 4 D) 1
- ii) The exact solution of the system of equation  $10x + y + z = 12$ ,  $x + 10y + z = 12$ ,  $x + y + 10z = 12$  by inspection is equal to A)  $(-1, 1, 1)$  ; B)  $(1, 1, 1)$  ; C)  $(-1, -1, -1)$  ; D) None
- iii) If the given system of linear equations in 'n' variables is consistent then the number of linearly independent - solution is given by A) n ; B) n - 1 ; C) r - n ; D) n - r
- iv) The trivial solution for the given system of equations  $9x - y + 4z = 0$ ,  $4x - 2y + 3z = 0$ ,  $5x + y - 6z = 0$  is  
A)  $(1, 2, 0)$  B)  $(0, 4, 1)$  C)  $(0, 0, 0)$  D)  $(1, -5, 0)$ .
- b. Using elementary transformation reduce each of following matrices to the normal form,  $\begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & -1 & 2 & 5 \\ 3 & 1 & 1 & 8 \\ 2 & -2 & 3 & 7 \end{bmatrix}$ . (06 Marks)
- c. Test for consistency and solve the system,  $2x + y + z = 10$ ,  $3x + 2y + 3z = 18$ ,  $x + 4y + 9z = 16$ . (06 Marks)
- d. Apply Gauss-Jordan method to solve the system of equations,  $2x + 5y + 7z = 52$ ,  $2x + y - z = 0$ ,  $x + y + z = 9$  (04 Marks)
- 8 a. Choose your answers for the following : (04 Marks)
- i) A square matrix A is called orthogonal if, A)  $A = A^2$  B)  $A = A^{-1}$  C)  $AA^{-1} = I$  D) None
- ii) The eigen values of the matrix,  $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$  are A) 2, 3, 8 B) 2, 3, 9 C) 2, 2, 8 D) None
- iii) The eigen vector X of the matrix A corresponding to eigen value  $\lambda$  and satisfy the equation,  
A)  $AX = \lambda X$  B)  $\lambda(A - X) = 0$  C)  $XA - \lambda X = 0$  D)  $|A - \lambda I|X = 0$
- iv) Two square matrices A and B are similar if, A)  $A = B$ ; B)  $B = P^{-1}AP$ ; C)  $A' = B'$ ; D)  $A^{-1} = B^{-1}$
- b. Show that the transformation,  $y_1 = 2x_1 - 2x_2 - x_3$ ,  $y_2 = -4x_1 + 5x_2 + 3x_3$ ,  $y_3 = x_1 - x_2 - x_3$  is, regular and find the inverse transformations. (06 Marks)
- c. Diagonalize the matrix,  $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ . (06 Marks)
- d. Reduce the quadratic form,  $x_1^2 + 2x_2^2 - 7x_3^2 - 4x_1x_2 + 8x_2x_3$  into sum of squares. (04 Marks)