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Third Semester B.E. Degree Examination, June/July 2014

Discrete Mathematical Structures

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. For any three sets A, B, C, prove: $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$. **(06 Marks)**

b. Among the integers from 1 to 200, find the number of integers that are:
 i) not divisible by 5
 ii) divisible by 2 or 5 or 9
 iii) not divisible by 2 or 5 or 9. **(07 Marks)**

c. A problem is given to four students A, B, C, D whose chances of solving it are 1/2, 1/3, 1/4, 1/5 respectively. Find the probability that the problem is solved. **(07 Marks)**

2 a. Define a tautology and contradiction. Prove that, for any propositions p, q, r, the compound proposition $[(p \to q) \wedge (q \to r)] \to (p \to r)$ is a tautology. **(06 Marks)**

b. Define the dual of logical statement. Verify the principle of duality for the following logical equivalence: $[\neg(p \wedge q) \to \neg p \vee (\neg p \vee q)] \Leftrightarrow (\neg p \vee q)$. **(07 Marks)**

c. Define converse, inverse and contra-positive of a conditional with truth table. Write down the contra-positive of $[p \to (q \to r)]$ with:
 i) only one occurrence of the connective $\to$
 ii) no occurrence of the connective $\to$. **(07 Marks)**

3 a. Negate and simplify each of the following:
 i) $\exists x, [p(x) \vee q(x)]$
 ii) $\forall x, [p(x) \wedge \neg q(x)]$
 iii) $\forall x, [p(x) \to q(x)]$ **(06 Marks)**

b. Establish the validity of the following argument:

$$\begin{array}{l} \forall x, [p(x) \vee q(x)] \\ \underline{\forall x, [\{\neg p(x) \wedge q(x)\} \to r(x)]} \\ \therefore \forall x, [\neg r(x) \to p(x)] \end{array}$$

(07 Marks)

c. Prove that every even integer n with $2 \le n \le 26$ can be written as a sum of atmost three perfect squares. **(07 Marks)**

4 a. Let $a_0 = 1$, $a_1 = 2$, $a_2 = 3$ and $a_n = a_{n-1} + a_{n-2} + a_{n-3}$ for $n \ge 3$. Prove that $a_n \le 3^n$ for all positive integers n. **(06 Marks)**

b. Find an explicit definition of the sequence defined recursively by $a_1 = 7$, $a_n = 2a_{n-1} + 1$ for $n \ge 2$. **(07 Marks)**

c. The Lucas numbers are defined recursively by $L_0 = 2$, $L_1 = 1$ and $L_n = L_{n-1} + L_{n-2}$ for $n \ge 2$. Evaluate L_2 to L_{10}. **(07 Marks)**

PART – B

5 a. Suppose A, B, C $\subseteq$ Z X Z with A = $\{(x, y) | y = 5x - 1\}$; B = $\{(x, y) | y = 6x\}$; C = $\{(x, y) | 3x - y = -7\}$. Find: (i) $A \cap B$, (ii) $B \cap C$, (iii) $\overline{\overline{A} \cup \overline{C}}$, (iv) $\overline{B} \cup \overline{C}$. **(06 Marks)**

b. Define stirling number of second kind. Find the number of ways of distributing four distinct objects among three identical containers with some containers possibly empty. **(07 Marks)**

c. If $f : A \to B$, $g : B \to C$, and $h : C \to D$ are three functions then prove that $(h \circ g) \circ f = h \circ (g \circ f)$. **(07 Marks)**

6 a. Let A = {1, 2, 3, 4}, B = {w. x. y. z} and C = {5, 6, 7}. Also, let R_1 be a relation from A to B, defined by $R_1 = \{(1, x), (2, x), (3, y), (3, z)\}$ and R_2 and R_3 be relations from B to C, defined by $R_2 = \{(w, 5), (x, 6)\}$, $R_3 = \{(w, 5), (w, 6)\}$. Find $R_1 \circ R_3$. **(06 Marks)**

b. Find the number of equivalence relations that can be defined on a finite set A with $|A| = 6$. **(07 Marks)**

c. For A = {a, b, c, d, e}, the Hasse diagram for the poset (A, R) is as shown below:

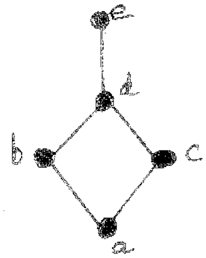

Fig.Q6(c)

i) Determine the relation matrix for R.
ii) Construct the diagraph for R. **(07 Marks)**

7 a. Define subgroup of a group. Let G be a group and let J = { $x \in G \mid xy = yx$ for all $y \in G$}. Prove that J is a subgroup of G. **(06 Marks)**

b. State and prove Lagrange's theorem. **(07 Marks)**

c. The word c = 1010110 is sent through a binary symmetric channel. If p = 0.02 is the probability of incorrect receipt of a signal, find the probability that c is received as r = 1011111. Determine the error pattern. **(07 Marks)**

8 a. The parity-check matrix for an encoding function $E : z_2^3 \to z_2^6$ is given by

$$H = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

i) Determine the associated generator matrix.
ii) Does this code correct all single errors in transmission? **(06 Marks)**

b. Prove that the set z with binary operations $\oplus$ and $\odot$ defined by $x \oplus y = x + y - 1$; $x \odot y = x + y - xy$ is a cumulative ring. **(07 Marks)**

c. Show that z_6 is not an integral domain. **(07 Marks)**

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