

**Fourth Semester B.E. Degree Examination, June/July 2014**

**Engineering Mathematics – IV**

Time: 3 hrs.

Max. Marks:100

**Note: Answer FIVE full questions, selecting at least TWO questions from each part.**

**PART – A**

- 1
  - a. Obtain a solution upto the third approximation of  $y$  for  $x = 0.2$  by Picard's method, given that  $\frac{dy}{dx} + y = e^x$ ;  $y(0) = 1$ . (06 Marks)
  - b. Apply Runge-Kutta method of order 4, to find an approximate value of  $y$  for  $x = 0.2$  in steps of 0.1, if  $\frac{dy}{dx} = x + y^2$  given that  $y = 1$  when  $x = 0$ . (07 Marks)
  - c. Using Adams-Bashforth formulae, determine  $y(0.4)$  given the differential equation  $\frac{dy}{dx} = \frac{1}{2}xy$  and the data,  $y(0) = 1$ ,  $y(0.1) = 1.0025$ ,  $y(0.2) = 1.0101$ ,  $y(0.3) = 1.0228$ . Apply the corrector formula twice. (07 Marks)
- 2
  - a. Apply Picard's method to find the second approximation to the values of 'y' and 'z' given that  $\frac{dy}{dx} = z$ ,  $\frac{dz}{dx} = x^3(y + z)$ , given  $y = 1$ ,  $z = \frac{1}{2}$  when  $x = 0$ . (06 Marks)
  - b. Using Runge-Kutta method, solve  $\frac{d^2y}{dx^2} - x\left(\frac{dy}{dx}\right)^2 + y^2 = 0$  for  $x = 0.2$  correct to four decimal places. Initial conditions are  $x = 0$ ,  $y = 1$ ,  $y' = 0$ . (07 Marks)
  - c. Obtain the solution of the equation  $\frac{2d^2y}{dx^2} = 4x + \frac{dy}{dx}$  at the point  $x = 1.4$  by applying Milne's method given that  $y(1) = 2$ ,  $y(1.1) = 2.2156$ ,  $y(1.2) = 2.4649$ ,  $y(1.3) = 2.7514$ ,  $y'(1) = 2$ ,  $y'(1.1) = 2.3178$ ,  $y'(1.2) = 2.6725$  and  $y'(1.3) = 3.0657$ . (07 Marks)
- 3
  - a. Define an analytic function in a region  $R$  and show that  $f(z)$  is constant, if  $f(z)$  is an analytic function with constant modulus. (06 Marks)
  - b. Prove that  $u = x^2 - y^2$  and  $v = \frac{y}{x^2 + y^2}$  are harmonic functions of  $(x, y)$  but are not harmonic conjugate. (07 Marks)
  - c. Determine the analytic function  $f(z) = u + iv$ , if  $u - v = \frac{\cos x + \sin x - e^{-y}}{2(\cos x - \cosh y)}$  and  $f(\pi/2) = 0$ . (07 Marks)
- 4
  - a. Find the images of the circles  $|z| = 1$  and  $|z| = 2$  under the conformal transformation  $w = z + \frac{1}{z}$  and sketch the region. (06 Marks)
  - b. Find the bilinear transformation that transforms the points  $0, i, \infty$  onto the points  $1, -i, -1$  respectively. (07 Marks)
  - c. State and prove Cauchy's integral formula and hence generalized Cauchy's integral formula. (07 Marks)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.  
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8-50, will be treated as malpractice.

**PART – B**

- 5 a. Obtain the solution of the equation  $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + \left(x^2 - \frac{1}{4}\right)y = 0$ . (06 Marks)
- b. Obtain the series solution of Legendre's differential equation,  

$$(1-x^2) \frac{d^2y}{dx^2} - 2x \frac{dy}{dx} + n(n+1)y = 0$$
 (07 Marks)
- c. State Rodrigue's formula for Legendre polynomials and obtain the expression for  $P_4(x)$  from it. Verify the property of Legendre polynomials in respect of  $P_4(x)$  and also find  $\int_{-1}^1 x^3 P_4(x) dx$ . (07 Marks)
- 6 a. Two fair dice are rolled. If the sum of the numbers obtained is 4, find the probability that the numbers obtained on both the dice are even. (06 Marks)
- b. Given that  $P(\bar{A} \cap \bar{B}) = \frac{7}{12}$ ,  $P(A \cap \bar{B}) = \frac{1}{6} = P(\bar{A} \cap B)$ . Prove that A and B are neither independent nor mutually disjoint. Also compute  $P(A/B) + P(B/A)$  and  $P(\bar{A}/\bar{B}) + P(\bar{B}/\bar{A})$ . (07 Marks)
- c. Three machines  $M_1$ ,  $M_2$  and  $M_3$  produces identical items. Of their respective outputs 5%, 4% and 3% of items are faulty. On a certain day,  $M_1$  has produced 25% of the total output,  $M_2$  has produced 30% and  $M_3$  the remainder. An item selected at random is found to be faulty. What are the chances that it was produced by the machine with the highest output? (07 Marks)
- 7 a. In a quiz contest of answering 'Yes' or 'No', what is the probability of guessing atleast 6 answers correctly out of 10 questions asked? Also find the probability of the same if there are 4 options for a correct answer. (07 Marks)
- b. Define exponential distribution and obtain the mean and standard deviation of the exponential distribution. (07 Marks)
- c. If X is a normal variate with mean 30 and standard deviation 5, find the probabilities that (i)  $26 \leq X \leq 40$ , (ii)  $X \geq 45$ , (iii)  $|X - 30| > 5$ . [Give that  $\phi(0.8) = 0.2881$ ,  $\phi(2.0) = 0.4772$ ,  $\phi(3.0) = 0.4987$ ,  $\phi(1.0) = 0.3413$ ] (06 Marks)
- 8 a. Certain tubes manufactured by a company have mean life time of 800 hrs and standard deviation of 60 hrs. Find the probability that a random sample of 16 tubes taken from the group will have a mean life time (i) between 790 hrs and 810 hrs, (ii) less than 785 hrs, (iii) more than 820 hrs. [ $\phi(0.67) = 0.2486$ ,  $\phi(1) = 0.3413$ ,  $\phi(1.33) = 0.4082$ ]. (06 Marks)
- b. A set of five similar coins is tossed 320 times and the result is:
- |               |   |    |    |     |    |    |
|---------------|---|----|----|-----|----|----|
| No. of heads: | 0 | 1  | 2  | 3   | 4  | 5  |
| Frequency:    | 6 | 27 | 72 | 112 | 71 | 32 |
- Test the hypothesis that the data follow a binomial distribution. [Given that  $\psi_{0.05}^2(5) = 11.07$ ] (07 Marks)
- c. It is required to test whether the proportion of smokers among students is less than that among the lectures. Among 60 randomly picked students, 2 were smokers. Among 17 randomly picked lecturers, 5 were smokers. What would be your conclusion? (07 Marks)

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