Engineering Maths 2 - June 2014

TOTAL MARKS: 100
TOTAL TIME: 3 HOURS

(1) Question 1 is compulsory.
(2) Attempt any **four** from the remaining questions.
(3) Assume data wherever required.
(4) Figures to the right indicate full marks.

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1 (a) Choose the correct answer: (4 marks)

(i) The general solution of the equation $x^2p^2+3xyp+2y^2=0$ is _______

$$(a)\ (y^2x-c)(xy-c)=0$$
$$(b)\ (x-y-c)(x^2+y^2-c)=0$$
$$(c)\ (xy-c)(x^2y-c)=0$$
$$(d)\ (y-x-c)(x^2+y^2+c)=0$$

ii) The given differential equation is solvable for y, if it is possible to express y in terms of _______
(a) y and p
(b) x and p
(c) x and y
(d) y and x

iii) The singular solution of Clairaut's equation is ________
(a) y=xg(x)+f[g(x)]
(b) y=cx+f(c)
(c) cy + f(c)
(d) y g^2 (x) + f [g(x)]

iv) The singular solution of the equation y=px-log p is ________

$$(a)\ y^2=4ax$$
$$(b)\ x=1-\log x$$
$$(c)\ y=1-\log\left(\frac{1}{x}\right)$$
$$(d)\ x^2=y\log x$$

1 (b) Solve p^2-2p sin h x-1=0 (4 marks)

1 (c) Solve y=2px+ $\tan^{-1}(xp^2)$ (6 marks)

1 (d) Obtain the general solution and singular solution of Clairaut's equation is (y-px)(p-1)=p (6 marks)

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2 (a) Choose the correct answer: (4 marks)

(i)The complementary function of $[D^4+4]x=0$ is ________

$$(a)\ x = e^{-4}[c_1 \cos t + c_2 \sin t] + e^1[c_3 \cot t + c_4 \sin t]$$
$$(b)\ x = [c_1 \cos t + c_2 \sin t] + [c_3 \cos t + c_4 \sin t]$$
$$(c)\ x = [c_1 + c_2 t]e^{-t}$$
$$(d)\ x = [c_1 + c_2 t]e^{t}$$

Find the particular integral of $(D^3-3D^2+4)y=e^{2x}$ is _______

$$(a)\ \frac{x^2 e^{2x}}{6}$$
$$(b)\ \frac{x^2 e^{3x}}{6}$$
$$(c)\ \frac{x^2 e^{x}}{6}$$
$$(d)\ \frac{x^2 e^{4x}}{6}$$

$$iii)\ Roots\ of\ \frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 0\ are\ \_\_\_\_\_\_\_\_\_$$
$$(a)\ 2 \pm i$$
$$(b)\ 3 \pm i$$
$$(c)\ 2 \pm 2i$$
$$(d)\ -2 \pm i$$

iv) Find the particular integral of $(D^3 + 4D)y=\sin 2x$ is ________

$$(a)\ \frac{x\sin x}{8}$$
$$(b)\ \frac{-x\sin x}{8}$$
$$(c)\ \frac{-x\sin 2x}{8}$$
$$(d)\ \frac{x\sin 2x}{8}$$

2 (b) (4 marks)

$$Solve\ \frac{d^2y}{dx^3}+6\frac{d^2y}{dx^2}+11\frac{dy}{dx}+6y=e^x+1$$

2 (c) (6 marks)

$$Solve\ \frac{d^2y}{dx^2}-4y=\cos h(2x-1)+3^x$$

2 (d) (6 marks)

$$Solve\ \frac{dy}{dx}+y=ze^x,\ \frac{dz}{dx}+z=y+e^x$$

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3 (a) Choose the correct answer: (4 marks)

(i) The Wronskian x and x e^x is ________

(a) e^x

(b) e^{2x}

(c) e^{-2x}

(d) e^{-x}

(ii) The complementary function of $x^2y''-xy'-3y=x^2 \log x$ is ________

(a) $c_1 \cos(\log x) + c_2 \sin(\log x)$

(b) $c_1 x^{-1}+c_2x$

(c) c_1x+c2x^3

(d) $c_1 \cos x + c_2 \sin x$

iii) To transform $(1+x)^2y''+(1+x)y'+y=2\sin\log(1+x)$ into a linear differential equation with constant coefficient ________

(a) $(1+x)=e^{t}$

(b) $(1+x)=e^{-t}$

(c) $(1+x)^2=e^{t}$

(d) $(1-x)^2=e^{t}$

iv) The equation $a_0(ax+b)^2y''+a_1(ax+b)y'+a_2y=\phi(x)$ is _________

(a) Simulataneous equation

(b) Cauchy's linear equation

(c) Legendre linear equation

(d) Euler's equation

3 (b) Using the variation of parameters method to solve the equation $y''+2y'+y=e^{-x}$ log x. (6 marks)

3 (c) (6 marks)

$$Solve\ x^2\frac{d^2y}{dx^2}-(2m-1)x\frac{dy}{dx}+(m^2+n^2)y=n^2x^m\log x$$

3 (d) Obtain the Frobenius method solve the equation (6 marks)

$$x\frac{d^2y}{dx^2}+\frac{dy}{dx}-y=0$$

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4 (a)i) Partial differential equation by eliminating a and b from the relation $Z=(x-a)^2+(y-b)^2$ is ________ (4 marks)

(a) $p^2q^2=4z$

(b) $pq=4z$

(c) $r=4z$

(d) $t=4$

ii) The Lagrange's linear partial differential equation $Pp+Qq=R$ the subsidiary equation is _______

$$(a)\ \frac{dx}{R}=\frac{dy}{P}=\frac{dz}{Q}$$

$$(b)\ \frac{dx}{P}=\frac{dy}{Q}=\frac{dz}{R}$$

$$(c)\ \frac{dx}{Q}=\frac{dy}{R}=\frac{dz}{P}$$

$$(d)\ \frac{dx}{P}+\frac{dy}{Q}+\frac{dz}{R}$$

iii) By the method mof separation of variable we seek a solution in the form is

____

(a) x=x+y
(b) z= x^2+y^2
(c) x=z+y
(d) x=x(x)y(y)

iv) The solution of

$$\frac{\partial^2 z}{\partial x^2} = \sin(xy) \ is$$

$$(a) \ z = -x^2 \sin(xy) + y \ f(x) + \phi(x)$$

$$(b) \ \frac{-\sin(xy)}{y} + x \ f(y) + \phi(y)$$

$$(c) \ z = \frac{-\sin xy}{x^2} + yf(x) + \phi(x)$$

$$(d) \ None \ of \ these$$

4 (b) From the partial differential equation of all sphere of radius 3 units having their centre in the xy-plane. (4 marks)

4 (c) (6 marks)

$$Solve \ x(y^2+z)p - y(x^2+z) = z(x^2-y^2)$$

4 (d) Use the method of seperation of variable to solve (6 marks)

$$y^3 \frac{\partial z}{\partial x} + x^2 \frac{\partial z}{\partial y} = 0$$

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5 (a) Choose the correct answer: (4 marks)

(i) The value of

$$\int_0^1 \int^{x^2} 0e^{y/x} dydx \ is \ \_\_\_\_\_\_$$

(a) 0
(b) 1
(c) 3
(d) 1/2

ii) The value of ? (1/2) is _______

$$(a)\ 2\sqrt{\pi}$$
$$(b)\ \pi/2$$
$$(c)\ \sqrt{\pi}$$
$$(d)\ \sqrt{2x}$$

(iii) The integral

$$\int_0^a \int_y^a \frac{x}{x^2+y^2} dxdy$$

after changing the order of integration is _______

$$(a)\ \int_0^a \int_0^x dydx$$
$$(b)\ \int_0^a \int_0^x \frac{x}{x^2+y^2} dxdy$$
$$(c)\ \int_0^x \int_0^x \frac{x}{x^2+y^2} dxdy$$
$$(d)\ \int_0^x \int_0^a \frac{x}{x^2+y^2} dxdy$$

iv) The value of β (3, 1/2) is ________

$$(a)\ \frac{15}{16}$$
$$(b)\ \frac{16}{15}$$
$$(c)\ \frac{16}{5}$$
$$(d)\ \frac{16}{3}$$

5 (b) Change the order of integration in (4 marks)

$$\int_0^{4a} \int_{\frac{x^2}{4x}}^{\sqrt{az}} dydx$$

and hence evaluate the same.

5 (c) (6 marks)

$$Evaluate \int_{-c}^{c}\int_{-b}^{b}\int_{-a}^{a}(x^2+y^2+z^2)dx\,dy\,dz$$

5 (d) (6 marks)

$$Prove\ that \int_0^1 \frac{x^2}{\sqrt{1-x^4}}dx^{-x}\int_0^1 \frac{1}{\sqrt{1+x}}dx = \frac{\pi}{4\sqrt{2}}$$

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6 (a) Let S be the closed boundary surface of a region of volume V then for a vector field of difined in V and S ∫$_S$ f.nds is ________ (4 marks)

(a) ∫$_V$ curl of dv
(b) ∫$_V$ f dv
(c) ∫$_V$ grad dv
(d) None of these

$$If \int_c f.dr\ where\ f = 3xy\hat{i} - y^2\hat{j}$$

and C is the part of the parabola y=2x^2 from the region (0, 0) to the point (1, 2) is

____

(a) 7/6
(b) -7/6
(c) 3x+3y
(d) -35

iii) In the Green's theorem in the plane

$$\oint_c Mdx + Ndy = \_\_\_\_\_\_\_\_$$

$$(a)\ \iint_R \left[\frac{\partial M}{\partial y} + \frac{\partial N}{\partial x}\right] dxdy$$

$$(b)\ \iint_R \left[\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right] dxdy$$

$$(c)\ \iint_R \left[\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right] dxdy$$

$$(d)\ \iint_R \left[\frac{\partial N}{\partial x} + \frac{\partial M}{\partial y}\right] dxdy$$

iv) A necessary and sufficient condition that the line integral

$$\int_c \vec{F}.d\vec{r}$$

for any closed curve C is ________

$$(a)\ div\vec{F} = 0$$
$$(b)\ div\vec{F}e0$$
$$(c)\ curl\vec{F} = 0$$
$$(d)\ grad\vec{F} = 0$$

6 (b) Using the divergence theorem, evaluate (4 marks)

$$\int_c f.nds\ where\ f = 4xz\hat{i} - y^2\hat{j} + yz\hat{k}$$

and S is the surface of the cube bounded by x=0, x=1, y=0, y=1, z=0, z=1.

6 (c) Use the Green's theorem, evaluate (6 marks)

$$\iint_c (2x^2 - y^2)dx + (x^2 + y^2)dy$$

where C is the triangle formed by the lines x=0, y=0 and x+y=1.

6 (d) Verify the Stoke's theorem for (6 marks)

$$f = -y^3\hat{i} + x^3\hat{j}$$

where S is the circle disc $x^2+y^2 \leq 1$, z=0.

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7 (a) Choose the correct answer: (4 marks)

$$(i)\ L\{\sinh at\} = \_\_\_\_\_\_\_\_$$
$$(a)\ \frac{s}{s^2 - a^2}$$
$$(b)\ \frac{s}{s}s^2 + a^2$$
$$(c)\ \frac{a}{s^2 - a^2}$$
$$(d)\ \frac{a}{s^2 + a^2}$$

ii) if L{f(t)}=F(s) then L{e^{at}f(t)} is _______ (a) F(s+a)
(b) F(s-a)
(c) F(s)
(d) None of these

$$iii)\ L\left\{\frac{e^t \sin t}{t}\right\}$$

$$(a)\ \frac{\pi}{2} + \tan^{-1}(s-1)$$

$$(b)\ \frac{\pi}{2} + \tan^{-1} s$$

$$(c)\ \frac{\pi}{2} - \cot^{-1} s$$

$$(d)\ \cot^{-1}(s-1)$$

iv) Transform of unit step function L{u(t-a)} is, ______

$$(a)\ \frac{e^{as}}{s}$$

$$(b)\ \frac{e^{-s}}{s}$$

$$(c)\ \frac{e^{2s}}{s}$$

$$(d)\ \frac{e^{-as}}{s}$$

7 (b) (4 marks)

$$Evaluate\ L\left\{3^t + \frac{\cos 2t - \cos 3t}{t} + t\sin t\right\}$$

7 (c) Find the Laplace transform of the triangular wave, given by (6 marks)

$$f(t) = \begin{cases} t & 0 \\ 2C-1 & C<1<2C \end{cases} \quad and\ f(t+2c) = f(t)$$

7 (d) (6 marks)

$$Express\ f(t) = \begin{cases} \cos t & if 0 < 1 < \pi \\ \cos 2t & if \pi < t < 2\pi \\ \cos 3t & if\ t > 2\pi \end{cases}$$

in terms of unit step function and hence find L{f(t)}

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8 (a) Choose the correct answer: (4 marks)

$$L^{-1}\left\{\cot^{-1}\left(\frac{s}{a}\right)\right\} = \_\_\_\_\_\_\_\_$$

$$(a)\ \frac{\sin t}{t}$$

$$(b)\ \frac{\sin at}{t}$$

$$(c)\ \frac{\sinh at}{t}$$

$$(d)\ \frac{\sinh t}{t}$$

$$(ii)\ L^{-1}\left\{\frac{1}{4s^2 - 36}\right\} = \_\_\_\_\_\_\_\_$$

$$(a)\ \frac{\cosh 6t}{4}$$

$$(b)\ \frac{\sin 3t}{12}$$

$$(c)\ \frac{\sinh 3t}{12}$$

$$(d)\ \frac{\cosh 3t}{6}$$

$$iii)\ L^{-1}\left\{\frac{1}{s(s^2 + a^2)}\right\} = \_\_\_\_\_\_\_\_$$

$$(a)\ \frac{1 - \cos at}{a^2}$$

$$(b)\ \frac{1 + \cos at}{a^2}$$

$$(c)\ \frac{1 - \sin at}{a^2}$$

$$(d)\ \frac{1 + \sin 3t}{6}$$

$$(iv)\ L^{-1}\left\{\frac{s^2-3s+4}{s^4}\right\} = \_\_\_\_\_\_\_\_\_\_$$

$$(a)\ 1-3t+2t^3$$

$$(b)\ 1+\frac{t^2}{3}$$

$$(c)\ t+\frac{3}{2}+1$$

$$(d)\ t-\frac{3}{2}t^2+\frac{2}{3}t^3$$

8 (b) (4 marks)

$$Find\ L^{-1}\left\{\frac{3s+7}{s^2-2s-3}\right\}$$

8 (c) Using Convolution theorem evaluate (6 marks)

$$L^{-1}\left\{\frac{1}{(s+1)(s^2+4)}\right\}$$

8 (d) (6 marks)

$$Solve\ \frac{d^2y}{dt}+5\frac{dy}{dt}+6y=5e^{2t}\ given\ that\ y(0)=2,\ \frac{dy(0)}{dt}=1$$

by using Laplace transform method.