Engg Mathematics 4 - December 2012

TOTAL MARKS: 100
TOTAL TIME: 3 HOURS

(1) Question 1 is compulsory.
(2) Attempt any **four** from the remaining questions.
(3) Assume data wherever required.
(4) Figures to the right indicate full marks.

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1 (a) Using the Taylor's series method, solve the initial value problem (6 marks)

$$\frac{dy}{dx} = x^2y - 1, y(0) = 1$$

at the point x=0.1

1 (b) Employ the fourth order Runge-Kutta method to solve (7 marks)

$$\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2} y(0) = 1$$

1 (c) (7 marks)

$$Given\ \frac{dy}{dx} = xy + y^2,\ y(0) = 1,\ y(0.1) = 1.1169,\ y(0.2) = 1.2773,\ y(0.3) = 1.5049$$

formula twice.

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2 (a) Exmploying the Picard's method, obtain the second order approximate solution of the following problem at x=0.2. (6 marks)

$$\frac{dy}{dx} = x + yz,\ \frac{dz}{dx} = y + zx,\ y(0) = 1,\ z(0) = -1$$

2 (b) Using the Runge-kutta method, find the solution at x=0.1 of the differential equation (7 marks)

$$\frac{d^2y}{dx^3} - x^2\frac{dy}{dx} - 2xy = 1$$

2 (c) Using the Milne's method, obtain an approximate solution at the point x=0.4 of the problem (7 marks)

$$\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} - 6y = 0,\ y(0) = 1,\ y'(0) = 0.1$$

y(0.3)=1.29865, y'(0.3)=1.873.

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3 (a) If f(z)=u+iv is an analytic function then prove that (6 marks)

$$\left[\frac{\partial}{\partial x}|f(z)|\right]^2 + \left[\frac{\partial}{\partial y}|f(z)|\right]^2 = |f'(z)|^2$$

3 (b) Find an analytic function whose imaginary part is $v=e^x\{(x^2-y^2)\cos y-2xy\sin y\}$ (7 marks)

3 (c) If f(z)=u(r,?)+ iv(r,?) is an analytic function, show that u and v satisfy the equation (7 marks)

$$\frac{\partial^2\varphi}{\partial r^2} + \frac{1}{r}\frac{\partial\varphi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\varphi}{\partial\theta^2} = 0$$

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4 (a) Find the bilinear transformation that maps the points 1, i, -1 onto the point i, 0, -i respectively. (6 marks)

4 (b) Discuss the transformation $W=e^x$. (7 marks)

4 (c) Evaluate (7 marks)

$$\int_C \frac{\sin\pi z^2 + \cos\pi z^3}{(z-1)^2(z-2)}dz$$

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5 (a) Express the polynomial $2x^3-x^2-3x+2$ in terms of Legendre polynomials. (6 marks)

5 (b) Obtain the series solution of Bessel's differential equation (7 marks)

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - n^2)y = 0$$

5 (c) Derive Rodrique's formula (7 marks)

$$P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2 - 1)^n.$$

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6 (a) State the axioms of probability. For any two events A and B, Prove that P(A?B)=P(A)+P(B)-P(A?B). (6 marks)

6 (b)A bag contains 10 white balls and 3 red ball while another bag contains 3 white balls and 5 red balls. Two balls are drawn at random from the first bag and put in the second bag and then a ball is drawn at random from the second bag. What is the probability that it is a white ball? (7 marks)

6 (c) In a bolt factory there are four machines A, B, C, D manufacturing respectively 20%, 15%, 25%, 40% of the total production. Out of these 5%, 4%, 3% and 2% respectively are defective. A bolt is drawn at random from the production and is found to be defective. Find the probability that it was manufactured by A or D. (7 marks)

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7 (a) The probability distribution of finite random variable x is given by the following table: (6 marks)

x_i	-2	-1	0	1	2	3
$p(x_i)$	0.1	k	0.2	2k	0.3	k

Determine the value of k and find the mean, variance and standard deviation.

7 (b) The probability that a pen manufactured by a company will be defective is 0.1. If 12 such pens are selected, find the probability that (i) exactly 2 will be defective. (ii) at least 2 will be defective, (iii) none will be defective. (7 marks)

7 (c) In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and standard deviation, given that A(0.5)=0.19 and A(1.4)=0.42, where A(z) is the area under the standard normal curve from O to z>0. (7 marks)

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8 (a)A biased coin is tossed 500 times and head turns up 120 times. Find the 95% confidence limits for the propertion of heads turning up in infinitely many tosses. (Given that z_c=1.96) (6 marks)

8 (b)A certain stimulus administered to each of 12 patients resulted in the following changes in blood pressure; (7 marks)

5, 2, 8, -1, 3, 0, 6, -2, 1, 5, 0, 4 (in appropriate unit)

Can it be conclude that, on the whole, the stimulus will change the blood pressure.

Use $t_{0.05}(11)=2.201$.

8 (c)A die is thrown 60 times and the frequency distribution for the number appearing on the face x is given by the following table: (0 marks)

Test the hypothesis that the die is unbiased. (Given that $x^2_{0.05}(5)=11.07$ and $x^2_{0.01}(5)=15.09$)