UNIVERSITY OF PUNE

[4362]-220

Electrical/Instrumentation/Computer/I.T.

S. E. Examination - 2013

Engineering Mathematics - III

(2008 Pattern)

Total No. of Questions : 12 **[Total No. of Printed Pages :6]**

[Time : 3 Hours] **[Max. Marks : 100]**

Instructions :

(1) Answer Q1 or Q2, Q3 OR Q4, Q5 OR Q6, From section I and Q7 OR Q8, Q9 OR Q10, Q11 OR Q12 From section II.

(2) Answers to the ***two sections*** *should be written in* ***separate answer-books****.*

(3) Neat diagrams must be drawn wherever necessary.

(4) Black figures to the right indicate full marks.

(5) Use of logarithmic tables, slide rule, Mollier charts, electronic pocket calculator and steam tables is allowed.

(6) Assume suitable data, if necessary.

SECTION-I

Q1. (a) Solve (any three) [12]

1)$(D^2-1)y = cosx\ coshx$

2) $(D^2+2D+D)y = e^{-x}logx$

3) $\frac{d^2y}{dx^2}+\frac{1}{x}\frac{dy}{dx} = 2+logx$

4) $\frac{dx}{xy^3-2x^4} = \frac{dy}{2y^4-x^3y} = \frac{dz}{9z(x^3-y^3)}$

Q.1 (b) An inductor of 0.25 henries is connected in series with a capacitor of 0.04 farads and a generator having alternative voltage given by 12sin10t. Find the charge and current at any time t. [5]

OR

Q2. (a) Solve: (any three) [12]

(1) $(D^2+1)y = x\cos 2x$
(2) $(D^2-2D+2)y = x^2+e^{-x}$
(3) $(D^2-2D)y = e^x \sin^x$ (variation of parameters)
(4) $((2x+5)^2\frac{d^2y}{dx^2}+8y-4(2x+5)\frac{dy}{dx} = 5\log(2x+5)$

Q2. (b) Solve: [5]

$$\frac{dx}{dt}+\frac{dy}{dt}-3x-y=e^t;\frac{dx}{dt}+2x+y=0$$

Q3. (a) If f(z) is analytic, prove that $(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2})\ |f(z)|^2 = 4\ |f^1(z)\ |^2$ [5]

(b) Show that the transformation $w= z+\frac{1}{z}-2i$ maps the circle $|z|=2$ an ellipse. [5]

(c) Evaluate: $\oint_c \frac{z+4}{z^2+2z+5}dz$ where c: $|z+2i| = \frac{3}{2}$ [6]

OR

Q4. (a) If f(z) = u+iv is analytic function, find f(z) if $u+v=3(x+y)+\frac{x-y}{x^2+y^2}$ [5]

(b) Find the bilinear transformation which maps the points 0,1,2 of z-plane to the points $1,\frac{1}{2},\frac{1}{3}$ of w plane respectively.

(c) Evaluate:

$$\int_o^{2\pi}\frac{d\theta}{5-3\cos\theta}$$ [6]

Q5. (a) Find fourier transform of

$$f(x)=\begin{cases} \cos x+\sin x & |x|\le\pi \\ o & |x|>\pi \end{cases}$$ [5]

(b) using fourier integral representation, show that :

$$\frac{2}{\pi}\int_{o}^{\infty}\frac{(\lambda^2+2)cos\lambda x}{\lambda^4+4}\,d\lambda = e^{-x}cosx, x > 0$$ [6]

(c) find z-transform of (any two) [6]

1) f(k)= $\frac{sinak}{k}, k \geq ,0$

2) f(k)= $k^2, k \geq 0$

3) f(k)= $\begin{cases} 7^k & k < 0 \\ 5^k & k \geq 0 \end{cases}$

OR

Q6. (a) Find inverse Z-transform of: (Any two) [8]

1) $\frac{2z^2-10z+13}{(z-3)^2(z-2)}$ $2 < |z| < 3$

2) $\frac{z(z+1)}{z^2-2z+1}$ $|z| > 1$

3) $\frac{z^2}{z^2+4}$ inversion integral method

b) Solve:

f(k)-4f(k-2)= $\left(\frac{1}{2}\right)^k$, $4 \geq 0$ [4]

c) Solve integral equation:

$$\int_0^{\infty} f(x)sin\lambda x dx = \frac{e^{-a\lambda}}{\lambda}, \lambda > 0$$ [5]

SECTION II

Q.7 (a) Following are the marks of ten students in math's- III and strength of material (SOM) calculate the coefficient of correlation. [8]

M-III	23	28	42	17	26	35	29	37	16	46
SOM	25	22	38	21	27	39	24	32	18	44

(b) Calculate the first four central moments and β_1, β_2 for the following distribution. [9]

x	0	1	2	3	4	5	6	7	8
F	1	8	28	56	70	56	28	8	1

OR

Q8. (a) The mean and varience of Binomial distribution are 6 and 2 respectively

Find: 1) $p(r \leq 1)$ 2) $p(r \geq 2)$ [6]

(b) If the probability that an individual suffers a bad reaction from a certain injection is 0.001, then determine the probability that out of 2000 individuals

1) Exactly 3 will suffer a bad reaction

2) More than 2 will suffer a bad reaction [6]

(c) A manufacturer of envelops knows that the weight of envelope is normally distributed with mean 1.9 gm and varience 0.01gm. find how many envelopes weighing

1) 2 grams or more

2) 2.1 grams or more

Can be expected in a given packet of 1000 envelopes (Given Area for z=1 is 0.3413 and Area for z=2 is 0.4772) [5]

Q.9 (a) If $\overline{r}(t) = t^2\overline{i} + t\overline{j} - 2t^3\overline{k}$ then [5]

Evaluate $\int_1^2 \overline{r} \times \frac{d^2\overline{r}}{dt^2} dt$

(b) Prove the following (any two) [6]

1) $\overline{b} \times \nabla[\overline{a} \cdot \nabla log r] = \frac{\overline{b}\times\overline{a}}{r^2} - 2\frac{(\overline{a}.\overline{r})(\overline{b}\overline{r})}{r^4}$

2) $\nabla^2\left(\frac{\overline{a}\cdot\overline{b}}{r}\right) = 0$

3) $\nabla \times (\frac{\overline{a}\times\overline{r}}{r}) = \frac{\overline{a}}{r} + \frac{(\overline{a}\cdot r)\overline{r}}{r^3}$

Q9. (c) Find the directional derivative of $\phi = 4xz^3 - 3x^2y^2z$ at (2,-1,2) in direction towards the point (2,-2,4) [5]

OR

Q10. (a) Verify whether $\overline{F} = (ysinz - sinx)\overline{i} + (xsinz + 2yz)\overline{j} + (xycosz + y^2)\overline{k}$ is irrotational and if so find the scalar ϕ such that $\overline{F} = \nabla\phi$ [5]

(b) If $\overline{u}$ and $\overline{v}$ are irrotational vectors then prove that $\overline{u} \times \overline{v}$ is solenoidal vector. [5]

(c) If directional derivate of $\phi = ax^2y + by^2z + cz^2x$. at (1,1,1) has maximum magnitude 15 in the direction parallel to $\frac{x-1}{2} = \frac{y-3}{-2} = \frac{z}{1}$
Then find values of a,b,c. [6]

Q11. (a) Find the work done in moving the particle long the curve $x = acos\theta$, $y = asin\theta, z = b\theta$ from $\theta = \frac{\pi}{4}$ to $\theta = \frac{\pi}{2}$ under the field of force given by
$\overline{F} = -3asin^2\theta\ cos\theta\overline{i} + a(2sin\theta - 3sin^3\theta)\overline{j} + bsin2\theta\overline{k}$ [5]

(b) Evaluate $\int\int_s (\nabla \times \overline{F}) \cdot \hat{n}\ ds$ where [6]

$\overline{F} = (x^3 - y^3)\overline{i} - xyz\,\overline{j} + y^3\overline{k}$ And S is the surface $x^2 + 4y^2 + z^2 - 2x = 4$ above the plane x=0.

(c) Evaluate $\int\int_s (x^3\overline{i} + y^3\overline{j} + z^3\overline{k}) \cdot\ d\overline{s}$ where S is the surface of the sphere $x^2 + y^2 + z^2 = 16$ [6]

Q.12 (a) Evaluate $\int\int_S \frac{\bar{r}}{r^3} \cdot \hat{n}\, ds$ by using Gauss Divergence theorem [5]

(b) Use Stoke's theorem to evaluate [6]

$\int_c (4y\bar{i} + 2z\bar{j} + 6y\bar{k}) \cdot d\bar{r}$ where 'c' is the curve of intersection of $x^2 + y^2 + z^2 = 2z$ and $x = z - 1$

(c) Two of the maxwell's equation are $\nabla \cdot \bar{B} = 0$, $\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$. given $\bar{B}$ = curl $\bar{A}$ then deduce that $\bar{E} + \frac{\partial \bar{A}}{\partial t} = -$grad (v) where V is a scalar point function.