Con. 8055-13. **GX-10120**

(Revised course)

(3 Hours) **[Total Marks : 80**

N.B. : (1) Question No. 1 is **compulsory**.
(2) Answer any **three** questions from question nos. 2 to 6.
(3) **Figures** to the **right** indicate **full** marks.
(4) Programming Calculators are **not allowed**.

1. (a) Evaluate $\int_0^{\infty} x^2\, 7^{-4x^2}\, dx$ **3**
(b) Solve $(D^4+4)y = 0$ **3**
(c) Prove that $E\,\nabla = \Delta = \nabla E$ **3**
(d) Solve $(x + 2y^3)\dfrac{dy}{dx} = y$. **3**
(e) Evaluate $\iint_R r^3\, dr\, d\theta$ over the region between the circles $r = 2\sin\theta$, $r = 4\sin\theta$. **4**
(f) Evaluate $\int_0^1 \int_y^{\sqrt{y}} \dfrac{x}{(1-y)\sqrt{y-x^2}}\, dy\, dx$ **4**

2. (a) Solve :- $(x^3y^4 + x^2y^3 + xy^2 + y)\, dx + (x^4y^3 - x^3y^2 - x^2y + x)\, dy = 0$ **6**
(b) Change the order of integral and hence evaluate $\int_0^5 \int_{2-x}^{2+x} dx\, dy$ **6**
(c) Prove that $\int_0^1 \dfrac{x^2}{\sqrt{1-x^4}}\, dx \int_0^1 \dfrac{dx}{\sqrt{1+x^4}} = \dfrac{\pi}{4\sqrt{2}}$ **8**

3. (a) Evaluate $\int_0^1 \int_{y^2}^1 \int_0^{1-x} x\, dx\, dy\, dz$. **6**
(b) Find the area of one loop of the lemniscate $r^2 = a^2 .\cos 2\theta$ **6**
(c) Solve $(D^3+2D^2+D)y = x^2e^{3x} + \sin^2 x + 2^x$. **8**

4. (a) Show that the length of arc of the parabola $y^2 = 4ax$ cut off by the line $3y = 8x$ is $a\left(\log 2 + \dfrac{15}{16}\right)$ **6**
(b) Using the method of variation of parameters solve $\dfrac{d^2y}{dx^2} + 4y = \tan 2x$. **6**
(c) Compute y(0.2) given $\dfrac{dy}{dx} + y + xy^2 = 0$, $y(0) = 1$ by taking $h = 0.1$ using Runge-Kutta method of fourth order correct to 4 decimals. **8**

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5. (a) Solve $\frac{dy}{dx}+x(x+y)=x^3(x+y)^3-1$. 6

(b) Solve $\frac{dy}{dx}-2y=3e^X, y(0)=0$ using Taylor series method. Find approximate value of y for x = 1 and 1.1. 6

(c) Evaluate $\int_0^6 \frac{dx}{1+x}$ using 8

(i) Trapezoidal rule

(ii) Simpson's $\left(\frac{1}{3}\right)^{rd}$ rule and

(iii) Simpson's $\left(\frac{3}{8}\right)^{th}$ rule.

Compare result with exact values.

6. (a) The current in a circuit containing an inductance L, registance R and voltage E sin wt is given by 6

$$L\frac{di}{dt}+Ri=E\sin wt$$

If i = 0 at t = 0, find i.

(b) Evaluate $\iint_R e^{2x-3y}\,dxdy$ over the triangle bounded by $x+y=1,\ x=1, y=1$. 6

(c) (i) Find the volume of solid bounded by the surfaces $y^2=4ax$, $x^2=4ay$ and the planes Z = 0, Z = 3. 4

(ii) Change to polar co-ordinates and evaluate 4

$$\int_0^a\int_{\sqrt{ax-x^2}}^{\sqrt{a^2-x^2}}\frac{dxdy}{\sqrt{a^2-x^2-y^2}}$$