

FE (SEM I) (REV) (CBSSGS) May 2014
applied mathematics I 13/05/14

(REVISED COURSE)

Q P Code : NP-17690

(3 Hours)

[Total Marks : 80]

- N.B. : (1) Questions No. 1 is compulsory.
(2) Attempt any three from the remaining questions.
(3) Assume suitable data if necessary.

1. (a) Prove that $\text{Sech}^{-1}(\sin\theta) = \log\left(\cot\frac{\theta}{2}\right)$ 3
- (b) If $x = \cos\theta - r\sin\theta$, $y = \sin\theta + r\cos\theta$ 3
prove that $\frac{dr}{dx} = \frac{x}{r}$
- (c) If $x = e^y \sec u$, $y = e^y \tan u$ 3
find $J\left(\frac{u, v}{x, y}\right)$
- (d) If $y = \sin px + \cos px$ 3
Prove that $y_n = p^n [1 + (-1)^n \sin 2px]^{\frac{1}{2}}$
- (e) Find the series expansion of $\log(1+x)$ in powers of x . Hence prove that 4
 $\log x = (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \dots$
- (f) If 'A' is skew-symmetric matrix of odd order then prove that it is singular. 4
2. (a) Show that the roots of the equation $(x+1)^6 + (x-1)^6 = 0$ are given by 6
 $-i \cot\left(\frac{2n+1}{12}\pi\right), n=0,1,2,3,4,5.$
- (b) Find two non-singular matrices P & Q such that PAQ is in normal form where 6
$$A = \begin{bmatrix} 1 & 2 & 3 & -4 \\ 2 & 1 & 4 & -5 \\ -1 & -5 & -5 & 7 \end{bmatrix}$$

Also find rank of A.
- (c) If $x + y = 2e^\theta \cos\phi$, $x - y = 2ie^\theta \sin\phi$ & u is a function of x & y the prove that 8
$$\frac{\partial^2 u}{\partial \theta^2} + \frac{\partial^2 u}{\partial \phi^2} = 4xy \frac{\partial^2 u}{\partial x \partial y}$$

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3. (a) Find the value of λ for which the equations $x_1 + 2x_2 + x_3 = 3$, $x_1 + x_2 + x_3 = \lambda$, $3x_1 + x_2 + 3x_3 = \lambda^2$ has a solution & solve them completely for each value of λ . 6
- (b) Divide 24 into three parts such that the product of the first, square of the second & cube of the third is maximum. 6
- (c) (i) If $\operatorname{cosec}\left(\frac{\pi}{4} + ix\right) = u + iv$ prove that $(u^2 + v^2)^2 = 2(u^2 - v^2)$ 4
- (ii) Prove that $\tan\left(i \log\left(\frac{a - ib}{a + ib}\right)\right) = \frac{2ab}{a^2 - b^2}$ 4
4. (a) Show that $\frac{\partial(u, v)}{\partial(x, y)} = 6r^3 \sin 2\theta$ given that $u = x^2 - y^2$, $v = 2x^2 - y^2$ & $x = r \cos\theta$, $y = r \sin\theta$. 6
- (b) If $\alpha = 1 + i$, $\beta = 1 - i$ & $\cot \theta = x + 1$ prove that $(x + \alpha)^n + (x + \beta)^n = (\alpha + \beta) \cos n\theta \operatorname{cosec}^n \theta$. 6
- (c) Using Gauss-seidel method, solve the following system of equations upto 3rd iteration. 8
- $$\begin{aligned} 5x - y &= 9 \\ -x + 5y - z &= 4 \\ -y + 5z &= -6 \end{aligned}$$
5. (a) Using De-Moivre's theorem, prove that 6
- $$\frac{\sin 6\theta}{\sin \theta} = 16 \cos^4 \theta - 16 \cos^2 \theta + 3$$
- (b) Expand $\frac{x}{e^x - 1}$ in powers of x . 6
- Hence prove that $\frac{x}{2} \left[\frac{e^x + 1}{e^x - 1} \right] = 1 + \frac{1}{12} x^2 - \frac{1}{720} x^4 + \dots$
- (c) If $y = \frac{\sin^{-1} x}{\sqrt{1 - x^2}}$ 8
- prove that $(1 - x^2)y_{n+2} - (2n + 3)xy_{n+1} - (n + 1)^2 y_n = 0$. Hence find $y_n(0)$

6. (a) Examine the linear dependence or independence of vectors $(1, 2, -1, 0)$, $(1, 3, 1, 3)$, $(4, 2, 1, -1)$ & $(6, 1, 0, -5)$ 6

- (b) If $u = f\left(\frac{x-y}{xy}, \frac{z-x}{xz}\right)$ prove that 6

$$x^2 \frac{\partial u}{\partial x} + y^2 \frac{\partial u}{\partial y} + z^2 \frac{\partial u}{\partial z} = 0$$

- (c) (i) Fit a straight line to the following data with x - as independent variable. 4

X : 1965 1966 1967 1968 1969

Y : 125 140 165 195 200

- (ii) Evaluate $\lim_{x \rightarrow 0} (1 + \tan x)^{\cot x}$ 4
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