

(REVISED COURSE) QP Code : NP-17762

(3 Hours)

[Total Marks : 80

- N.B. : (1) Question No.1 is compulsory.
(2) Attempt any three question from remaining five questions.

1. (a) Evaluate $\int_0^1 \sqrt{\sqrt{x}-x} dx$ 3
- (b) Solve $[D^4 - 4D^3 + 8D^2 - 8D + 4]y = 0$ 3
- (c) Prove that $(1 + \Delta)(1 - \nabla) = 1$ 3
- (d) Change to polar co-ordinate and evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} (x^2+y^2) dy dx$ 3
- (e) Solve $(x^2 - 4xy - 2y^2) dx + (y^2 - 4xy - 2x^2) dy = 0$ 4
- (f) Evaluate $\int_0^a \int_0^{\sqrt{1+x^2}} \frac{1}{1+x^2+y^2} dy dx$ 4
2. (a) Solve $xy(1+xy^2) \frac{dy}{dx} = 1$ 6
- (b) Change the order of integration and evaluate $\int_0^\infty \int_0^x x e^{-x^2/y} dy dx$ 6
- (c) Evaluate $\int_0^{\pi/2} \frac{dx}{a^2 \sin^2 x + b^2 \cos^2 x}$ and show that 8
- $$\int_0^{\pi/2} \frac{dx}{(a^2 \sin^2 x + b^2 \cos^2 x)^2} = \frac{\pi}{4ab} \left(\frac{1}{a^2} + \frac{1}{b^2} \right)$$
3. (a) Evaluate $\iiint x^2 yz dx dy dz$ throughout the volume bounded by $x=0, y=0, z=0,$ 6
 $x+y+z=1.$
- (b) Find the area bounded by parabola $y^2=4x$ and the line $y=2x-4.$ 6
- (c) Use the method of variation of parameter to solve 8
- $$\frac{d^2y}{dx^2} + 5 \frac{dy}{dx} + 6y = e^{-2x} \sec^2 x (1 + 2 \tan x)$$

4. (a) Find the total length of the loop of the curve $9y^2 = (x + 7)(x + 4)^2$. 6
- (b) Solve $\frac{d^2y}{dx^2} + 2y = x^2e^{3x} + e^x - \cos 2x$ 6
- (c) Apply Runge-kutta method of fourth order to find an approximate value of y at $x = 0.2$ 8
if $\frac{dy}{dx} = x + y^2$ give that $y = 1$, when $x = 0$ in step of $h = 0.1$.
5. (a) Solve $y(x^2y + e^x) dx - e^x dy = 0$. 6
- (b) Using Taylor's series method solve $\frac{dy}{dx} = 1 - 2xy$ given that $y(0) = 0$ and hence find 6
 $y(0.2)$ and $y(0.4)$.
- (c) Compute the value of the definite integral $\int_{0.2}^{1.4} (\sin x - \log_e x + e^x) dx$, by 8
- (i) Trapezoidal Rule
- (ii) Simpson's one third Rule
- (iii) Simpson's three-eighth Rule.
6. (a) The motion of a particle is given by $\frac{d^2x}{dt^2} = -k^2x - 2h\frac{dx}{dt}$, solve the equation when 6
 $h = 5, k = 4$ taking $x = 0, v = v_0$ at $t = 0$. Show that the time of maximum displacement is independent of the initial velocity.
- (b) Evaluate $\iint (x^2 + y^2) dx dy$ over the area of triangle whose vertices are $(0, 0), (1, 0),$ 6
 $(1, 2)$.
- (c) Find the volume bounded by $y^2 = x, x^2 = y$ and the planes $z = 0$ and $x + y + z = 1$. 8