S.E. Electronics Sem III / Exam

Con. 3029-10. Engineering Mathematics III **AN-2492**

(3 Hours) **[Total Marks : 100**

N.B. (1) Question No. **1** is **compulsory**.
(2) Answer any **four** question out of the remaining **six** questions.
(3) **Figures** to **right** indicate **full** marks.

1. (a) Prove that : 5

$$\int_0^\infty e^{-st}\left(\frac{\sin at + \sin bt}{t}\right) dt$$

$$= \pi - \tan^{-1}\left(\frac{s(a+b)}{ab - s^2}\right)$$

(b) Find Fourier transform of 5

$$f(x) = 1 - x^2, \ |x| \le 1$$
$$= 0 \quad , \ |x| > 1$$

(c) If A is nonsingular square matrix of order n, then show that : 5

$$|\text{adj adj } A| = |A|^{(n-1)^2}.$$

(d) Find the bilinear transform which maps the points $Z = (1, -1, \infty)$ onto the points $\omega = (1 + i, 1 - i, 1)$. 5

2. (a) Evaluate the following : 8

(i) $\int_0^\infty e^{-t} \sin^5 t \, dt$

(ii) $L\left\{ e^{-u} \int_0^t e^u \cos hu \, du \right\}$

(b) Express the function :— 6

$$f(x) = -e^{kx}, \ x < 0$$
$$= e^{-kx}, \ x > 0$$

as Fourier Integral and hence prove that $\int_0^\infty \frac{\omega \sin \omega x}{\omega^2 + k^2} = \frac{\pi}{2} e^{-kx}$

if $x > 0$, $k > 0$.

(c) Find Z-transform of $f(k) = \sin \alpha k$, $k \ge 0$ where α is real. 6

3. (a) If f(z) is analytic function, prove that : 8

(i) $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) = 4 \frac{\partial^2}{\partial z \, \partial \bar{z}}$

(ii) $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) \log |f'(z)| = 0$

(b) Find Inverse Z-transform of $\frac{3z^2 + 2z}{z^2 - 3z + 2}$ for $| < |z| < 2$ 6

(c) Find complex form of Fourier Series for :— 6

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(c) Find complex form of Fourier Series for :— 6

$$f(x) = 0, \; 0 < x < l$$
$$= a, \; l < x < 2l.$$

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4. (a) Use Laplace transform to solve $\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 8y = 1$ where $y(0) = 0$, $y^1(0) = 1$. 8

(b) If $f(z) = u + iv$ is analytic and $u - v = e^x (\cos y - \sin y)$, find $f(z)$ in terms of z. 6

(c) Show that $\overline{F} = (ye^{xy} \cos z)\, i + (xe^{xy} \cos z)\, j - (e^{xy} \sin z)k$ is irrotational and find the scalar potential for $\overline{F}$. 6

5. (a) Find Fourier expression of $f(x) = x^2$ in $-\pi \le x \le \pi$ and hence prove that : 8

(i) $\frac{\pi^2}{6} = \sum_1^\infty \frac{1}{n^2}$

(ii) $\frac{\pi^2}{12} = \sum_1^\infty \frac{(-1)^{n+1}}{n^2}$

(iii) $\frac{\pi^4}{90} = \sum_1^\infty \frac{1}{n^4}$

(b) State convolution theorem for Laplace transform and hence find : 6

$$L\left\{\frac{s+2}{(s^2+4s+8)^2}\right\}.$$

(c) Using Laplace transform, evaluate $\int_0^\infty e^{-2t} (1 + 2t - 3t^2 + 4t^3)\, H(t-1)\, dt$. 6

6. (a) If A and B are matrices given below, reduce A to normal form and find rank of A and hence find rank of $3A^2 - AB$: 8

$$A = \begin{bmatrix} 0 & 2 & 1 & 2 \\ 0 & 2 & 1 & 1 \\ 2 & 6 & 3 & 5 \\ 2 & 4 & 2 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 6 & 3 & 6 \\ 0 & 4 & 3 & 3 \\ 6 & 18 & 7 & 15 \\ 6 & 12 & 6 & 10 \end{bmatrix}$$

(b) Find the work done in moving the particle along the curve $r = (e^t \cos t)\, i + (e^t \sin t)\, j$ from the point (1,0) to $(e^{2\pi}, 0)$ in the force field $\overline{F} = \frac{xi + yj}{(x^2 + y^2)^{3/2}}$. 6

(c) Find the image of the circle $|z| = k$ under the transformation $\omega = 3z + 4 + 2i$. 6

7. (a) Determine λ and μ for which the system **8**

$$3x - 2y + z - \mu = 0$$
$$5x - 8y + 9z - 3 = 0$$
$$2x + y + \lambda z + 1 = 0$$

have : (i) no solution

(ii) unique solution

(iii) Infinite number of solutions. Also find infinite solutions of the systems.

(b) Show that the following set of functions is orthogonal over $(-l, l)$ **6**

$$S = \left\{ 1, \cos\frac{n\pi x}{l}, \sin\frac{n\pi x}{l}, n \in \pi V \right\}.$$

(c) Using Green's theorem evaluate $\int_c \left(e^{x^2} - xy \right) dx - \left(y^2 - ax \right) dy$ where c is the circle $x^2 + y^2 = a^2$. **6**